New method for studying neutrino mixing and mass differences

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Abstract

A toy model shows how neutrino masses and mixing can be investigated by studying the behavior of a radioactive ion which decays by K-capture BEFORE and DURING its weak decay by K-capture. A new oscillation phenomenon providing information about neutrino mixing is obtained by following the ion before and during the decay. This normally neglected process is shown to be consistent with quantum mechanics and causality. Measuring the oscillation without detecting the neutrino avoids losses in conventional experiments due to the low neutrino absorption cross section. The normally unobservable long wave lengths are made observable by having the radioactive source move a long distance circulating around in a storage ring. The initial ion wave packet has a momentum spread required by Heisenberg and contains pairs of components with different momenta and energies. These can produce neutrino amplitudes in two mass eigenstates with different momenta which mix to produce a single ν_e state. In this typical quantum mechanics "two-slit" or "which path" experiment a transition between the same initial and final states can go via two paths in energy-momentum space with a phase difference producing interference and oscillations.

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I. INTRODUCTION

A. The two principal difficulties of neutrino experiments

A recent experiment [1] describes an oscillation observed in the decay of a radioactive ion before and during the emission of an unobserved neutrino. This phenomenon offers a new and very interesting method for determining neutrino masses and mixing angles [2,3].

1. Ordinary neutrino oscillation experiments are difficult because
    - The neutrino absorption cross section is tiny. The number of neutrino events actually used in ordinary experiments is many orders of magnitude smaller than the number events creating the neutrinos.
    - The oscillation wave lengths are so large that it is difficult to actually follow even one oscillation period in any experiment.
2. This experiment opens up a new line for dealing with these difficulties
    - The oscillation is measured without detecting the neutrino. Detection of every neutrino creation event avoids the losses from the low neutrino absorption cross section.
    - The long wave length problem is solved by having the radioactive source move a long distance circulating around in a storage ring. The data if correct show many oscillations in the same experiment.

This paper considers the basic quantum mechanics of the first difficulty and shows in a toy model that it is possible in principle to observe and measure neutrino oscillations by looking only at the radioactive source. The second difficulty warrants further investigation.

The theoretical analysis in this paper was motivated by discussions with Paul Kienle at a very early stage of the experiment in trying to understand whether the effect was real or just an experimental error.

B. The K-capture experiment

The original version of this paper was written on the basis of private information before the release of ref. [1] and considered the decay of a nucleus by emitting an electron into the atomic K-shell.

A similar anaysis can be applied to a K-capture experiment in which a radioactive ion in an atom or ion decays by capturing an electron from the K-shell or other atomic shell and emits a monoenergetic neutrino. Here there are a number of initial states having different degrees of ionization. Interference can only occur between initial states having the same degree of ionization. Otherwise the analysis is the same as for the K-emission experiment. However, we note that the paths in space for a neutral atom are not easily influenced by external electromagnetic fields that can otherwise influence the path or orbit in space taken by the ion in the initial state.

The emitted electron-neutrino ν_e is now known to be a linear combination of several neutrino mass eigenstates. If the initial state has a definite momentum and energy, the conservation of energy and momentum determines the energy and momentum of the neutrino and therefore its mass. This is then a "missing mass" experiment in which the mass of the neutrino is determined without the observation of the neutrino. Interference between amplitudes from different neutrino mass states cannot be observed in such a missing-mass experiment.

At first it seems rather peculiar that neutrino oscillations can be observed in the state of a radioactive ion before its decay into an unobserved neutrino. One wonders about causality and how the initial ion can know how it will decay. But much discussion and thought revealed that the essential quantum mechanics is a "two-slit" or "which-path" experiment [4] in momentum space which preserves causality because no information about the final state is available to the initial ion.

It is not a missing mass experiment because the initial ion wave function is a wave packet containing a combination of momenta which prevent it from being used in a missing mass experiment. Its well-defined relative phases are determined by its localization in space. These relative phases change with time in accordance with the relative energy differences in the packet.

The weak decay transition then produces a final neutrino via any of its momentum eigenstates. Since the same final state can be produced by any of the momentum components in the initial wave function, the path in energy-momentum space between the initial and final states is not known and the corresponding amplitudes can be coherent and interfere.

The relative phases in the initial wave function are independent of the final state, which is created only at the decay point. Thus there is no violation of causality. No information about the final state exists before the decay. Although time-dependent perturbation theory might suggest that a decay amplitude can be present before the decay, the continued observation of the initial ion before the decay rules out any influence of any final state amplitude on the decay process. It is like the "Schroedinger cat" experiment in which the door is always open so that there is a continuous measurement of whether the cat is still alive.

C. The quantum mechanics of realistic experiments

In any realistic experiment the Heisenberg uncertainty principle prevents the momentum of the initial state from being known with sufficient precision to determine the neutrino mass. The GSI experiment [1] observed periods of modulation of the order of 7 seconds with ions traveling at 71% of the velocity of light. The ions thus travel a distance of $0.71 \cdot 3 \cdot 10^5 \cdot 7 \approx 10^6$ kilometers in a single period of oscillation. The uncertainty in knowing the position of the experiment within the laboratory is tiny incomparison with this enormous oscillation wave length. Heisenberg then tells us that the momentum uncertainty required to produce thes oscillations is equally tiny in comparison to the momentum fluctuations required by confining the experiment within the laboratory.

$$\frac{\delta x}{\lambda_{osc}} \approx \frac{\delta p_{osc}}{\delta p_{loc}} \ll 1 \tag{1.1}$$

where δx denotes the uncertainty in the position of the experiment in the laboratory, λ_{osc} denotes the oscillation wave length, δp_{osc} denotes the momentum difference required for these oscillations and δp_{loc} the momentum difference in the initial state required by its localization in the laboratory. Thus this is not a missing-mass experiment.

The momentum difference between the different neutrino mass states is thus much smaller than the momentum uncertainly required by Heisenberg from knowing that the experiment takes place within the laboratory. The initial state is a wave packet in momentum space containing the different momenta required to produce decays to neutrino mass eigenstates with different masses. The transition to the final ν_e state can therefore go via different neutrino mass eigenstates with no record of which mass eigenstate produced the final ν_e. The contributions via different neutrino mass eigenstates define different paths in momentum space which are not observed in the experiment. The contributions to the final state amplitude via these different paths are therefore coherent and interference between them can be observed producing oscillations.

D. Coherence and decoherence

Understanding coherence and decoherence is crucial here. All the relevant physics is in the initial state of the ion. The amplitude at the decay point is the coherent sum of the amplitudes from all allowed paths in energy-momentum space. But coherence between amplitudes is not introduced by simple ignorance of which path was taken [5]. Coherence results only from an uncertainty required by quantum mechanics. Most nuclear and particle theorists are unfamiliar with investigations on coherence and decoherence in condensed matter and mesoscopic physics [6,7]. In neutrino oscillation experiments the answer is clear. The neutrino is detected, time is not measured and the detector has a momentum uncertainty. The relevant neutrino states are those of the same energy and different momentum. These are the only pairs of states where coherence can be preserved.

This experiment [1] is completely different. Time is measured and a time dependence is the crucial new ingredient in the experiment. However, the preparation of the initial state is complicated and considerable thought has to be devoted to some questions raised in the paper [1]. A radioactive ion is trapped in a storage ring with a circumference of 108.3 m and a revolution frequency of about 2 MHz. Time oscillations in the radioactive decay by emission of an unobserved neutrino are observed with a period of about 10 seconds. How could coherence be preserved over the time span of some ten seconds? What is the effect of the continuous monitoring of the state of the ion?

Condensed matter theorists have examined coherence and decoherence in many contexts. There are also the concepts of "preselection" and "postselection" introduced and extensively explored by Yakir Aharonov and collaborators.

The purpose of this paper is to describe the basic quantum mechanics in a toy model where the initial radioactive ion is moves in a straight line and oscillations are observed as a function of time. This toy model is highly unrealistic. A precise analysis of the real experiment enabling the determination of neutrino mass differences from the observed oscillation periods in a storage ring is left for further investigation. It is much more complicated than this toy model.

The toy model provides some insight into how the coherence is preserved and the effect of continuous monitoring. In the model the initial state is a wave packet of plane waves which is moving in space and time. But plane waves have a constant amplitude over all space and have an equal probability of being behind the moon as in the laboratory where the state is created. The relative phases of the individual plane wave components must be adjusted so that there is no probability of finding the particle outside the laboratory at this time. The center of mass momentum of the packet can change appreciably in the preparation and cooling of the state. But these interactions cannot suddenly create a probability that the ion has jumped to behind the moon. The limits on the size of the wave packet in space preserve the relative phase of neighboring components with the tiny difference in momenta relevant to the observed oscillations.

II. THE QUANTUM MECHANICS OF OSCILLATION EXPERIMENTS

A. No coherence in a missing mass experiment

A radioactive ion that decays by K-capture emits a neutrino which is a linear combination of neutrino mass eigenstates. If the energy and momentum of the initial ion and also the recoil momentum of the final ion are known the energy and momentum of the emitted neutrino are determined and therefore its mass. This would suggest that there can be no coherence nor oscillations between the neutrino states.

To see that this argument misses the exciting observable physics from the beta-decay experiment we examine the analogous case of the emission of an electron and a neutrino in the decay of a pion at rest. [4,8]

When a pion decays at rest $\pi \rightarrow e\nu$ the energies E_e, E_ν and momenta $\vec{p}_e, \vec{p}_\nu$ of the electron and neutrino can all be known. This is then just a "Missing Mass" experiment. The neutrino mass M_ν is uniquely determined by $M_\nu^2 = (M_\pi - E_e)^2 - p_e^2$. So how can there be coherence and interference between states of different mass? We are guided to the resolution of this paradox by experience in condensed matter physics discussing which amplitudes are coherent in quantum mechanics [6,9,10].

B. Why it is not a missing mass experiment

The original Lederman-Schwartz-Steinberger experiment found that the neutrinos emitted in a $\pi - \mu$ decay produced only muons and no electrons. Experiments now show that at least two neutrino mass eigenstates are emitted in $\pi - \mu$ decay and that at least one of them can produce an electron in a neutrino detector. The experimentally observed absence of electrons can be explained only if the electron amplitudes received at the detector from different neutrino mass eigenstates are coherent and exactly cancel. This implies that sufficient information was not available to determine the neutrino mass from energy and momentum conservation. A missing mass experiment was not performed.

Coherence or interference between different neutrino mass eigenstates cannot be observed in a “missing mass” experiment where the mass of an unobserved neutrino is uniquely determined by other measurements and momentum and energy conservation.

The resolution of these contradictions is just simple quantum mechanics. In any experiment which can detect neutrino oscillations, the position of the source must be known with an error much smaller than the wave length of the oscillation to be observed. The quantum mechanical uncertainty principle therefore forces coherence between neutrino mass eigenstates having the same energy and different momenta. Stodolsky’s theorem [9] states that in an experiment which does not explicitly measure time the quantum mechanical density matrix for the system is diagonal in energy and there can be interference between states of different energy and no explicit time dependence in a correct theoretical description. The location in space already says it all.

III. THE K-CAPTURE EXPERIMENT IN A TOY MODEL

A. The basic theory

The experiment describes the decay of a radioactive ion into another radioactive ion by K-capture and the emission of a neutrino. Since there are two different neutrino mass states, the decay has two neighboring channels for decay. The standard theoretical description here is a linear combination of two Breit-Wigner amplitudes. In a conventional experiment in which the momentum and energy of the initial ion is known, the momentum and energy of the recoil ion can be measured, the neutrino mass is determined and there can be no oscillations.

We have seen that in pion decay the localization of the pion in space produces an uncertainty in momentum that prevents a determination of the neutrino mass. The absence of a time measurement requires [9] that only states of the same energy can be coherent.

In the K-capture experiment [1] the localization of the experiment in the laboratory also produces quantum momentum fluctuations. But many other factors are very different and much more complicated:

1. Time is measured and interference between states of different energies can be observed.
2. The initial state is a single-particle state with a well defined mass. There is therefore an uncertainty in energy required by the energy-momentum relation for a particle with a definite mass. The broadening of the mass value by the small decay width is neglected here.
3. Oscillations are observed in the initial state, not the final state, as a result of the time dependence of relative phases in the initial wave function. The experiment is a "which-path experiment" because which particular momentum eigenstate in the initial wave function produced the neutrino is not known.
4. The final neutrino is not observed. It is known to have been created as an electron neutrino which is a well-defined linear combination of two or three mass eigenstates. Further measurements on the final neutrino cannot affect the oscillations.
5. The initial state is moving in a circular orbit in a magnetic field. The kinematics may not be simple because the vector potential of the magnetic field complicates the definition of momentum and momentum conservation and can also introduce Aharonov-Bohm phases normally neglected.
6. Lorentz transformations to bring the momentum of the initial ion to an approximate rest system are not simple because the direction of the Lorentz tranformation must change with the motion of the ion around the ring.

In this paper the crucial question of which amplitudes are coherent is considered in the framework of a toy model.

B. The K-capture experiment as a “Two-slit” experiment

This model demonstrates that oscillations can occur and shows how an analysis of a realistic experiment requires a detailed investigation of what can be measured, what can not and where coherence can occur. The relation between the observed interference pattern and the neutrino masses is determined by which terms are coherent. The kinematics of the toy model is examined by noting the following conditions for coherence.

1. The initial state is a wave packet where Heisenberg prevents the measurement of its energy and momentum.
2. The final neutrino is a coherent combination of two neutrino mass states.
3. The toy model used here assumes that momentum and energy of the final recoil ion is observable, whether it is measured or not. Therefore states with different recoil momentum and energy cannot be coherent.
4. An open remaining question is the possible energy non-conservation in times short compared to the time necessary to resolve the two neutrino energy states. In this case the final state has an entangled wave function with two components having different recoil energies and different neutrino masses. This possibility is outside the framework of our present toy model but must be seriously considered for realistic cases.

The crucial ingredient is the inability to know the momentum of the initial state because we know where it is and Heisenberg tells us that this requires an uncertainty in its momentum.

The initial and final states of this experiment are well defined. The initial state is a radioactive ion in a wave packet which is confined to definite region of configuration space and therefore has a momentum spread. The final state is a recoiling atom and the ν_e linear combination of the neutrino mass eigenstates produced when an electron disappears in the weak interaction. The location in space of the initial and final states is well defined within normal experimental errors. We now have a direct analog to the two-slit or which-path experiment. Here the transition can go via any of the neutrino mass eigenstates. These define different paths between the initial and final states.

The initial state is a wave packet with neither a sharp momentum nor a sharp energy. The waves on the two paths thus overlap in momentum and energy. Coherence can be observed only if we do not know which paths contributed to the transition.

This requires that the two paths have the have the the same recoil momentum and energy, which are observable in the final state. The neutrino mass eigenstates have therefore different momenta which Heisenberg tells us must be unobservable. Since momentum is conserved in the transition, the different paths require different momenta for the radioactive ion in the initial state. The momentum spread in the initial wave functis sufficient in practical experiments to suppress all information on “which path” in momentum space was taken in the transition.

The final ν_e state is a linear combination of mass eigenstates with different energies and different momenta and a well defined phase. During the time interval between the creation of the initial state and its decay the relative phases between the energy eigenstate components of the initial wave function change linearly with the time. Thus the probability that the decay will take place to the final ν_e oscillates with time. The period of the oscillation depends upon the momentum and energy differences which in turn depend upon the mass differences between the mass eigenstates.

The experimental observation of these oscillations provides a new experimental method to obtain information about the neutrino mass differences and the mixing angles of the neutrino mass matrix.

C. The kinematics of the transition

Both energy and momenta are conserved for each component of the wave packet which has a momentum $\vec{P}$ and energy E in the initial state. The final state has a recoil ion with momentum denoted by $\vec{P}_R$ and energy E_R and a neutrino with mass m, energy E_ν and momentum $\vec{p}_\nu$. The energy release in the transition at rest, $M - M_R$ is denoted by Q. The conservation laws then require

$$E_R = E - E_\nu; \quad \vec{P}_R = \vec{P} - \vec{p}_\nu; \quad M^2 + m^2 - M_R^2 = Q \cdot [2M - Q] + m^2 = 2EE_\nu - 2\vec{P} \cdot \vec{p}_\nu \tag{3.1}$$

We neglect transverse momenta and set $\vec{P} \cdot \vec{p}_\nu \approx Pp_\nu$ where P and p_ν denote the components of the momenta in the direction of the incident beam. Then

$$Q \cdot [2M - Q] + m^2 = 2E(E_\nu - p_\nu) + 2(E - P)p_\nu = \frac{2Em^2}{E_\nu + p_\nu} + 2(E - P)p_\nu \tag{3.2}$$

This can be rearranged for further application to give

$$\frac{p_\nu}{P} - \frac{E_\nu}{E} = \frac{p_\nu(E - P) + (p_\nu - E_\nu)P}{PE} = \frac{Q \cdot [2M - Q] + m^2}{2PE} - \frac{m^2}{E_\nu + p_\nu} \cdot \left[\frac{1}{P} + \frac{1}{E}\right] \ll 1 \tag{3.3}$$

We are interested in the changes in the kinematic variables δp_ν, δP, δE_ν and δE produced by a small change $\Delta(m^2)$ in the squared neutrino mass:

$$\frac{\Delta(m^2)}{2} = E(\delta E_\nu) + (\delta E)E_\nu - P(\delta p_\nu) - (\delta P)p_\nu \tag{3.4}$$

We assume that the interference occurs between two neutrino states with the same energy and different momenta and that there is no change in the recoil momentum.

$$\delta E_\nu = 0; \quad \delta p_\nu = \delta P = \frac{E}{P} \cdot \delta E; \quad \frac{\delta p_\nu}{\delta E} = \frac{\delta P}{\delta E} = \frac{E}{P} \tag{3.5}$$

Combining eqs.(3.3), (3.4) and (3.5) then gives

$$\frac{\Delta(m^2)}{2\delta E} = E_\nu - P\frac{\delta p_\nu}{\delta E} - p_\nu\frac{\delta P}{\delta E} = -E \cdot \left[1 + \left\{\frac{p_\nu}{P} - \frac{E_\nu}{E}\right\}\right] \approx -E \tag{3.6}$$

The phase difference at a time t between states produced by the neutrino mass difference on the motion of the initial ion in the laboratory frame with velocity $V = (P/E)$ is

$$\delta\phi \approx -\delta E \cdot t \approx -\frac{\Delta(m^2)}{2E} \cdot t = -\frac{\Delta(m^2)}{2\gamma M} \cdot t \tag{3.7}$$

where γ denotes the Lorentz factor E/M. The period for $\delta\phi = -2\pi$ is

$$\delta t \approx \frac{4\pi\gamma M}{\Delta(m^2)} \tag{3.8}$$

In ref. [2] the period of modulation obtained was given as

$$\delta t_{IRK} = \frac{8\pi\gamma M_R}{\Delta(m^2)} \tag{3.9}$$

The ratio of these two values is

$$\frac{\delta t}{\delta t_{IRK}} \approx \frac{M}{2M_R} \approx \frac{1}{2} \tag{3.10}$$

Neither of these two values should be taken seriously because of essential features neglected in the simple models. Furthermore, the initial ion is not free but is constrained by electromagnetic forces confining it to an orbit in a storage ring. These forces introduce potential energies which may be important in the kinematics. They also complicate any Lorentz transformation from a "rest system" to the laboratory system. Simple attempts to include such effects have so far been unsatisfactory and are not considered here. A full calculation may be necessary including these confining forces.

One interesting feature of these two estimates is that they are within an order of magnitude of the result obtained from neutrino experiments. This requires the period to have a scale defined by the ratio of the mass of the ion to the difference between the squared masses of the two neutrino states. Other derivations and attempts to "correct" unjustified approximations destroy this scaling by introducing the very different energy scale of the neutrino energy.

The question arises of a possible additional phase proportional to the distance along the path. If we consider the motion along a straight line path, The total relative phase between waves differing by a momentum δP for traversing a distance X with velocity V is

$$\delta\phi_{str} \approx \delta P \cdot X - \delta E \cdot t \approx \left[\frac{E}{P} \cdot V - 1\right] \delta E \cdot t = 0 \tag{3.11}$$

But if the motion is in a circular orbit in a magnetic field with a frequency independent of the momentum there is no additional phase accumulated in the motion around the ring and eqs(3.8) and (3.10) apply. The question of the phase difference between states of different momenta along the two slightly different paths in a storage ring can thus play a crucial role here and depends upon the experimental conditions.

IV. CONCLUSIONS

A new oscillation phenomenon providing information about neutrino mixing is obtained by following the initial radioactive ion before and during the decay. The difficulties introduced in conventional neutrino experiments by the tiny neutrino absorption cross sections and the very long oscillaton wave lengths are avoided here. Measuring the decay time enables every neutrino event to be observed and counted without the necessity of observing the neutrino via the tiny absorption cross section. The confinement of the initial ion in a storage ring enables long wave lengths to be measured within the laboratory.

Coherence between amplitudes produced by the weak decay of a radioactive ion by the emission of neutrinos with different masses has been shown to follow from the localization of the initial radioactive ion within a space interval much smaller than the oscillation wave length. This coherence is observable in following the motion of the initial radioactive ion from its entry into the apparatus to its decay. The amplitude for production of a ν_e from several mass eigenstates depends upon the relative phases of the contributions from components in the inital wave function having different energies and momenta. These relative phases increase linearly with time and produce oscillations.

Observing the period of these oscillations gives information about the neutrino mass differences and the mixing angles of the neutrino mass matrix. Reliable detailed values for the relation between the observed oscillation period and neutrino mass differences are not obtained in the crude models so far considered. At this point the fact that the values obtained (3.8) and (3.9) are within an order of magnitude of consistency with values obtained [2] from neutrino oscillation experiments is encouraging.

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REFERENCES

[1] Yu.A. Litvinov, H. Bosch et al nucl-ex0801/2079
[2] A.N.Ivanov, R.Reda and P.Kienle nucl-th0801/2121
[3] Manfred Faber nucl-th0801/3262
[4] Harry J. Lipkin. hep-ph/0505141, Phys.Lett. B642 (2006) 366
[5] Kurt Gottfried, *Am. J. Phys.* 68 (2000) 143.
[6] Ady Stern, Yakir Aharonov and Yoseph Imry, Phys Rev. A41 (1990) 3436.
[7] Harry J. Lipkin, hep-ph/9907551, Physics Letters B 477 (2000) 195 and references therein.
[8] Harry J. Lipkin, hep-ph/9901399, in Proceedings of the Europhysics Neutrino Oscillation Workshop (NOW'98) 7-9 September 1998. Amsterdam. Published in http://www.nikhef.nl/pub/conferences/now98/.
[9] Leo Stodolsky, Phys. Rev. D58 (1998) 036006.
[10] B. Kayser, Phys. Rev. D 24 (1981) 110.