

Bounds on Lorentz and CPT Violation from the Earth-Ionosphere Cavity

Matthew Mewes

Physics Department, Marquette University, Milwaukee, WI 53201, U.S.A.

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Electromagnetic resonant cavities form the basis of many tests of Lorentz invariance involving photons. The effects of some forms of Lorentz violation scale with cavity size. We investigate possible signals of violations in the naturally occurring resonances formed in the Earth-ionosphere cavity. Comparison with observed resonances places the first terrestrial constraints on coefficients associated with dimension-three Lorentz-violating operators at the level of 10^{-20} GeV.

I. INTRODUCTION

Modern versions [1] of the classic Michelson-Morley [2] and Kennedy-Thorndike [3] experiments are among the most sensitive tests of Lorentz invariance, the symmetry behind special relativity. Typically these experiments search for minute changes in the resonant frequencies of electromagnetic cavities. High quality factors allow for precise tracking of the frequencies, giving extreme sensitivities to possible deviations from perfect Lorentz symmetry. However, the effects of some forms of Lorentz violation increase with wavelength. As a result, high sensitivities may be achieved using very-low-frequency resonances, such as those that naturally occur in the Earth-ionosphere cavity, despite their relatively low quality factors.

In this work, we consider signals of Lorentz violation that would appear in Schumann resonances, the lowest-frequency standing waves that form in the atmosphere [4, 5]. We obtain conservative bounds by comparison with observations [6]. The Earth's surface and ionosphere form a cavity of immense size, leading to resonances with very long wavelengths. The lowest-frequency resonances have wavelengths that are comparable to the circumference of the Earth and have frequencies as low as 8 Hz.

Violations of Lorentz invariance are described by the Standard-Model Extension (SME) [7, 8]. The SME is a theoretical framework that provides a basis for many experimental and theoretical studies of Lorentz violation [9, 10], including those involving atoms [11, 12], hadrons [13, 14], fermions [15–18], the Higgs boson [19], gravity [20], and photons [1, 21–25]. In addition to resonant-cavity experiments, searches for Lorentz violation in photons using the SME approach include astrophysical searches for vacuum birefringence [22–25] and dispersion [24, 25]. The goal of the SME is the characterization of all violations of Lorentz symmetry that are consistent with known physics using effective field theory [26]. While motivated in part by the possibility of spontaneous symmetry breaking in strings [27, 28], it encompassed violations with other origins [29–36]. Much of the work on Lorentz violation has focused on the minimal SME, which includes operators of renormalizable dimension in a flat spacetime. While nonrenormalizable operators and curved spacetimes are of general interest [8, 20, 23, 24, 30, 37], Schumann resonances are particu-

larly sensitive to the dimension-three *CPT*-odd Lorentz-violating operators of the minimal-SME photon sector.

In cavities, Lorentz violation can introduce frequencies that depend on the orientation of the cavity, signalling rotation violations, and dependence on velocity resulting from boost violations [25]. The quantity determining their sensitivity is the dimensionless fractional frequency shift $\delta\nu/\nu$. To date, cavity experiments have focused primarily on one class of violations, namely the dimension-four *CPT*-even Lorentz-violating operators of the minimal SME. The coefficients associated with these operators $(k_F^{(4)})^{\alpha\beta\gamma\delta}$ are dimensionless. Therefore, dimensional analysis suggests frequency shifts of the form $\delta\nu/\nu \sim (k_F^{(4)})^{\alpha\beta\gamma\delta}$, which implies little or no dependence on frequency. Consequently, there is little advantage to using low-frequency resonances. In contrast, the coefficients associated with the dimension-three operators $(k_{AF}^{(3)})_\kappa$ have mass-dimension one. Therefore, we naively expect shifts in frequency that depend on the ratio $(k_{AF}^{(3)})_\kappa/\nu$. Given that $\nu \sim 10^{-23}$ GeV for Schumann resonances, we naively expect sensitivities on the order of 10^{-23} GeV to $k_{AF}^{(3)}$ coefficients, assuming at least order-one sensitivity to $\delta\nu/\nu$. While not as sensitive as birefringence tests [24], the bounds obtained here represent the first terrestrial bounds on the dimension-three operators, providing a valuable check on existing astrophysical constraints.

The structural outline of this paper is as follows. Section II provides some basic theory behind our calculation. In Sec. III we derive modified wave equations and describe a numerical method of determining the effects of Lorentz violations on Schumann resonances. The results of our calculation are discussed in Sec. IV. Unless otherwise stated, we use the notation and conventions of Refs. [7, 24].

II. BACKGROUND

In this section we discuss the theory behind the calculation of the Earth-ionosphere resonances in the presence of dimension-three Lorentz violations. We begin by discussing the conductivity of the atmosphere and the Schumann resonances in the usual case. We then review the modified electrodynamics including the *CPT*-odd operators of the minimal SME.

A. Conductivity profile

Resonances in the Earth-ionosphere cavity are excited by a number of man-made and natural phenomena, lightning being a primary source. The surface of the Earth forms the lower boundary and, in our calculation, is treated as a perfect conductor. The ionosphere forms a lossy upper boundary with a finite conductivity profile that increases with altitude. The conductivity of the lower atmosphere can be approximated by a “knee”-model that separates into two layers with exponentially increasing conductivity [38]:

$$\sigma(r) \simeq \begin{cases} \infty & r < R, \\ \sigma_0 \exp \frac{r-r_0}{\xi_l} & R < r < r_0, \\ \sigma_0 \exp \frac{r-r_0}{\xi_u} & r_0 < r, \end{cases} \quad (1)$$

where r is the distance from the center of the Earth, and $R \simeq 6400 \text{ km} \simeq 3.2 \times 10^{22} \text{ GeV}^{-1}$ is the Earth radius. The lower layer is dominated by positive and negative ions. The upper layer approximates the bottom of the ionosphere, which is dominated by free electrons. The knee radius r_0 is the boundary between the layers, and σ_0 is the conductivity at this transition. In units where $c = \hbar = \epsilon_0 = \mu_0 = 1$, we adopt values $r_0 = 1.009R$, $\xi_l = 0.007R$, $\xi_u = 0.0005R$, and $\sigma_0 = 1.3R^{-1}$ in the numerical calculations that follow. This profile yields frequencies and quality factors that closely match the observed resonances in the Lorentz-invariant limit.

In the conventional case, both transverse-magnetic (TM) and transverse-electric (TE) modes may be excited in the cavity. However, TE modes oscillate in the radial direction, implying wavelengths comparable to the height of the ionosphere, yielding frequencies in the kHz range. In contrast, the lowest-frequency TM modes vary little in the radial direction but form standing waves that encircle the Earth. As a result, the wavelengths are comparable to the Earth’s circumference, yielding frequencies as low as 8 Hz. Our goal is to understand the effects of Lorentz violations on these low-frequency Schumann resonances.

B. Modified electrodynamics

The lagrangian governing electromagnetic waves, including dimension-three Lorentz-violating operators, is given by [7]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}(k_{AF}^{(3)})_{\kappa}A_{\lambda}F_{\mu\nu}, \quad (2)$$

where $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is the field strength. The resulting theory preserves the usual gauge symmetry, but violates CPT invariance. Lorentz and CPT violations are controlled by the constant coefficients $(k_{AF}^{(3)})_{\kappa}$, which include a pseudoscalar $(k_{AF}^{(3)})_0$ and pseudovector $\mathbf{k}_{AF}^{(3)}$. The $(k_{AF}^{(3)})_{\kappa}$ coefficients are assumed to be constant, which leads to energy-momentum conservation. More generally one can consider Lorentz-violating backgrounds

with spacetime variations. These types of violations are particularly important when considering Lorentz violations in curved spacetimes [8, 20]. However, in scenarios where violations originate in the moments shortly after the big bang, it is likely that any variation has expanded, leading to little fluctuation over experimentally relevant time and length scales. Consequently, the idea presented here probes $(k_{AF}^{(3)})_{\kappa}$ in our local neighborhood. Astrophysical tests rely on light that has propagated across much of the visible universe. So they test Lorentz invariance over much larger scales and could be drastically affected by variations in $(k_{AF}^{(3)})_{\kappa}$. Therefore, while current bounds from astrophysical searches for birefringence currently lie at the 10^{-42} GeV level [24], terrestrial tests provide an important complementary set of constraints.

The equations of motion resulting from Eq. (2) provide Lorentz-violating inhomogeneous Maxwell equations. We are primarily interested in harmonic solutions and consider electric and magnetic fields of the form $\mathbf{E}(t) = \mathbf{E}(\omega)e^{-i\omega t}$, $\mathbf{B}(t) = \mathbf{B}(\omega)e^{-i\omega t}$. Together with the usual homogeneous equations, we arrive at a Lorentz-violating electrodynamics with a modified Ampère law and a conventional Faraday law:

$$\begin{aligned} \nabla \times \mathbf{B} + i\omega \mathbf{E} - 2(k_{AF}^{(3)})_0 \mathbf{B} + 2\mathbf{k}_{AF}^{(3)} \times \mathbf{E} - \sigma \mathbf{E} &= \mathbf{J}_s, \\ \nabla \times \mathbf{E} - i\omega \mathbf{B} &= 0. \end{aligned} \quad (3)$$

Here we have included the usual source current and conductivity terms. These conventional source terms result if we assume usual coupling to matter. The source current is not needed in determining the resonances but could be used in modeling the effects of individual lightning strikes. The conductivity term is necessary to obtain realistic frequencies and quality factors, which are affected by the profile of the upper boundary and the losses it introduces.

III. CALCULATION

Our goal is to calculate the resonant frequencies that result from the modified Maxwell equations. We begin by expanding into spherical harmonics and deriving modified wave equations. A numerical calculation is used to estimate the resulting resonant frequencies. The results are discussed in the following section.

A. Wave equations

We begin our search for resonances by first expressing the modified Maxwell equations (3) in spherical coordinates using the helicity-basis and identities discussed in Appendix A. This involves writing the Maxwell equations in covariant form, then using relation (A5) to express them in terms of $\partial/\partial r$ and covariant angular-momentum ladder operators J_{\pm} . Dropping the source

current, the result is

$$0 = J_+ B_- + J_- B_+ + (\omega + i\sigma)rE_r + 2ir(k_{AF}^{(3)})_0 B_r + 2r(k_{AF}^{(3)})_+ E_- - 2r(k_{AF}^{(3)})_- E_+, \quad (4)$$

$$0 = \pm \frac{\partial}{\partial r} r B_{\pm} - J_{\pm} B_r + (\omega + i\sigma)rE_{\pm} + 2ir(k_{AF}^{(3)})_0 B_{\pm} \pm 2r(k_{AF}^{(3)})_r E_{\pm} \mp 2r(k_{AF}^{(3)})_{\pm} E_r, \quad (5)$$

$$0 = J_+ E_- + J_- E_+ - \omega r B_r, \quad (6)$$

$$0 = \pm \frac{\partial}{\partial r} r E_{\pm} - J_{\pm} E_r - \omega r B_{\pm}, \quad (7)$$

where E_r , B_r are the radial field components, and E_{\pm} , B_{\pm} are negative/positive helicity components, as discussed in the appendix. The components of $\mathbf{k}_{AF}^{(3)}$ are defined by $(k_{AF}^{(3)})_a = \hat{e}_a \cdot \mathbf{k}_{AF}^{(3)}$, where \hat{e}_a are the helicity basis vectors. The components E_{\pm} and B_{\pm} are referred to as spin-weighted functions with spin-weight- (± 1) , while E_r , B_r have a weight of zero. Spin weight and helicity are equivalent up to a sign. Eqs. (4) and (6) provide two scalar relations, while (5) and (7) have a spin weight of ± 1 .

We can expand the field components in spin-weighted spherical harmonics. These provide the generalization of the familiar spherical harmonics to spin-weighted functions. The expansion takes the form

$$E_r = \sum \frac{1}{r} E_{jm}^{(0E)} {}_0 Y_{jm}, \quad (8)$$

$$E_{\pm} = \sum \sqrt{\frac{j(j+1)}{2}} \frac{1}{r} (\mp E_{jm}^{(1E)} - i E_{jm}^{(1B)})_{\pm 1} Y_{jm}, \quad (9)$$

$$B_r = \sum \frac{1}{r} B_{jm}^{(0B)} {}_0 Y_{jm}, \quad (10)$$

$$B_{\pm} = \sum \sqrt{\frac{j(j+1)}{2}} \frac{1}{r} (\mp B_{jm}^{(1B)} - i B_{jm}^{(1E)})_{\pm 1} Y_{jm}, \quad (11)$$

where $E_{jm}^{(0E)}$, $E_{jm}^{(1E)}$, $E_{jm}^{(1B)}$, $B_{jm}^{(0B)}$, $B_{jm}^{(1B)}$, and $B_{jm}^{(1E)}$ are r -dependent field coefficients. They are associated with total-angular-momentum eigenmodes and have E -type parity, $(-1)^j$, or B -type parity, $(-1)^{j+1}$. The $\sqrt{j(j+1)/2}$ and $1/r$ factors in the expansions are for convenience.

Using these expansions, we can express the Maxwell equations in terms of $E_{jm}^{(0E)}$, $E_{jm}^{(1E)}$, $E_{jm}^{(1B)}$, $B_{jm}^{(0B)}$, $B_{jm}^{(1B)}$, and $B_{jm}^{(1E)}$. First, using the ladder operators J_{\mp} to lower/raise the spin- (± 1) relations (5) and (7) we arrive at six scalar Maxwell equations. We then use the spherical-harmonic expansions of the fields and the orthogonality relation (A3) to get relations between the six expansion coefficients. Some algebra yields three E -

parity equations and three B -parity equations:

$$0 = j(j+1)B_{jm}^{(1E)} - i(\omega + i\sigma)rE_{jm}^{(0E)} + r\mathcal{K}_{jm}^{(0E)}, \quad (12)$$

$$0 = \frac{\partial}{\partial r} B_{jm}^{(1E)} - i(\omega + i\sigma)E_{jm}^{(1E)} + \frac{1}{j(j+1)}\mathcal{K}_{jm}^{(1E)}, \quad (13)$$

$$0 = r\frac{\partial}{\partial r} B_{jm}^{(1B)} - B_{jm}^{(0B)} + i(\omega + i\sigma)rE_{jm}^{(1B)} + \frac{r}{j(j+1)}\mathcal{K}_{jm}^{(1B)}, \quad (14)$$

$$0 = j(j+1)E_{jm}^{(1B)} + i\omega r B_{jm}^{(0B)}, \quad (15)$$

$$0 = \frac{\partial}{\partial r} E_{jm}^{(1B)} + i\omega B_{jm}^{(1B)}, \quad (16)$$

$$0 = r\frac{\partial}{\partial r} E_{jm}^{(1E)} - E_{jm}^{(0E)} - i\omega r B_{jm}^{(1E)}, \quad (17)$$

where the Lorentz- and CPT -violating contributions have been collected into the field combinations

$$\begin{aligned} \mathcal{K}_{jm}^{(0E)} &= 2(k_{AF}^{(3)})_0 B_{jm}^{(0B)} + 2i|\mathbf{k}_{AF}^{(3)}|_m E_{jm}^{(1E)} \\ &\quad + 2|\mathbf{k}_{AF}^{(3)}|(j-1)\mathcal{C}_{jm} E_{(j-1)m}^{(1B)} \\ &\quad - 2|\mathbf{k}_{AF}^{(3)}|(j+2)\mathcal{C}_{(j+1)m} E_{(j+1)m}^{(1B)}, \end{aligned} \quad (18)$$

$$\begin{aligned} \mathcal{K}_{jm}^{(1E)} &= 2(k_{AF}^{(3)})_0 j(j+1)B_{jm}^{(1B)} \\ &\quad + 2i|\mathbf{k}_{AF}^{(3)}|_m E_{jm}^{(1E)} + 2i|\mathbf{k}_{AF}^{(3)}|_m E_{jm}^{(0E)} \\ &\quad + 2|\mathbf{k}_{AF}^{(3)}|(j^2 + 2j - 1)\mathcal{C}_{jm} E_{(j-1)m}^{(1B)} \\ &\quad + 2|\mathbf{k}_{AF}^{(3)}|(j^2 - 2)\mathcal{C}_{(j+1)m} E_{(j+1)m}^{(1B)}, \end{aligned} \quad (19)$$

$$\begin{aligned} \mathcal{K}_{jm}^{(1B)} &= -2(k_{AF}^{(3)})_0 j(j+1)B_{jm}^{(1E)} - 2i|\mathbf{k}_{AF}^{(3)}|_m E_{jm}^{(1B)} \\ &\quad - 2|\mathbf{k}_{AF}^{(3)}|(j+1)\mathcal{C}_{jm} E_{(j-1)m}^{(0E)} \\ &\quad + 2|\mathbf{k}_{AF}^{(3)}|_j \mathcal{C}_{(j+1)m} E_{(j+1)m}^{(0E)} \\ &\quad + 2|\mathbf{k}_{AF}^{(3)}|(j^2 + 2j - 1)\mathcal{C}_{jm} E_{(j-1)m}^{(1E)} \\ &\quad + 2|\mathbf{k}_{AF}^{(3)}|(j^2 - 2)\mathcal{C}_{(j+1)m} E_{(j+1)m}^{(1E)}, \end{aligned} \quad (20)$$

where $\mathcal{C}_{jm} = \sqrt{(j^2 - m^2)/(4j^2 - 1)}$. Here we take the angular-momentum quantization axis along the direction of $\mathbf{k}_{AF}^{(3)}$. Note that Eqs. (12)-(14) correspond to the modified spherical Ampère law and Eqs. (15)-(17) are the usual Faraday law.

In the conventional case, where all coefficients for Lorentz violation are zero, rotational symmetry implies that resonances are eigenmodes of angular momentum with definite j and m values. The symmetry also implies degeneracy in m . So the different resonant frequencies correspond to different values of the total-angular-momentum index j . Setting $\mathcal{K}_{jm}^{(0E)} = \mathcal{K}_{jm}^{(1E)} = \mathcal{K}_{jm}^{(1B)} = 0$ in Eqs. (12)-(17), we also note that the Lorentz-invariant case splits according to parity. The B -parity resonances correspond to the high-frequency TE modes, while low-frequency TM Schumann resonances are the E -parity modes.

Allowing for Lorentz violations, the new symmetries of the system lead to several generic predictions. Rotational symmetry is preserved in the event that we have only isotropic violations associated with coefficient $(k_{AF}^{(3)})_0$.

This implies that the indexing and degeneracies of the modes is the same, but the frequencies may change. In contrast, the vector $\mathbf{k}_{AF}^{(3)}$ breaks the usual degeneracy. The system remains symmetric under rotations about $\mathbf{k}_{AF}^{(3)}$. So we expect resonances that are eigenmodes of these rotations with eigenvalues m , as usual. However, these coefficients break spherical symmetry, implying the index j is no longer associated with resonances. As a result, the usual $2j+1$ degeneracies should break, yielding modes with definite m but indefinite j . Consequently, we expect two types of effects that would signify possible Lorentz violation. One is a split of degeneracies leading to additional resonant frequencies. This results

from anisotropic violations. The other effect is a shift in frequencies that may result from either anisotropic and isotropic violations.

The above first-order differential equations can be reduced to second-order modified wave equations. We begin by using the Faraday law, Eqs. (15)-(17), to eliminate the magnetic field. We also use Eq. (12) to eliminate the electric field component $E_{jm}^{(0E)}$ in favor of $E_{jm}^{(1E)}$, $E_{jm}^{(1B)}$, and $\mathcal{K}_{jm}^{(0E)}$. The result of this process is three coupled equations relating the three sets of field components $E_{jm}^{(1E)}$, $E_{jm}^{(1B)}$, and $\mathcal{K}_{jm}^{(0E)}$:

$$0 = \frac{\partial^2}{\partial r^2} E_{jm}^{(1E)} + \frac{p_j^2}{\omega+i\sigma} \left(\frac{\partial}{\partial r} \frac{\omega+i\sigma}{p_j^2} \right) \frac{\partial}{\partial r} E_{jm}^{(1E)} + p_j^2 E_{jm}^{(1E)} + \frac{ip_j^2}{\omega+i\sigma} \frac{\partial}{\partial r} \frac{1}{rp_j^2} \mathcal{K}_{jm}^{(0E)} + 2i|\mathbf{k}_{AF}^{(3)}| \frac{m\omega}{(\omega+i\sigma)j(j+1)} \mathcal{K}_{jm}^{(0E)} - 2(k_{AF}^{(3)})_0 \frac{p_j^2}{(\omega+i\sigma)\omega} \frac{\partial}{\partial r} E_{jm}^{(1B)} - 2|\mathbf{k}_{AF}^{(3)}| \frac{p_j^2 m}{(\omega+i\sigma)j(j+1)} E_{jm}^{(1E)} + 2|\mathbf{k}_{AF}^{(3)}| \frac{m}{(\omega+i\sigma)r} \frac{\partial}{\partial r} E_{jm}^{(1E)} + 2i|\mathbf{k}_{AF}^{(3)}| \frac{p_j^2(j^2+2j-1)\mathcal{C}_{jm}}{(\omega+i\sigma)j(j+1)} E_{(j-1)m}^{(1B)} + 2i|\mathbf{k}_{AF}^{(3)}| \frac{p_j^2(j^2-2)\mathcal{C}_{(j+1)m}}{(\omega+i\sigma)j(j+1)} E_{(j+1)m}^{(1B)}, \quad (21)$$

$$0 = \frac{\partial^2}{\partial r^2} E_{jm}^{(1B)} + p_j^2 E_{jm}^{(1B)} - 2|\mathbf{k}_{AF}^{(3)}| \frac{m\omega}{j(j+1)} E_{jm}^{(1B)} + 2(k_{AF}^{(3)})_0 \frac{(\omega+i\sigma)\omega}{p_j^2} \frac{\partial}{\partial r} E_{jm}^{(1E)} + 2i(k_{AF}^{(3)})_0 \frac{\omega}{p_j^2 r} \mathcal{K}_{jm}^{(0E)} - 2i|\mathbf{k}_{AF}^{(3)}| \frac{(j^2+2j-1)\mathcal{C}_{jm}\omega}{j(j+1)} E_{(j-1)m}^{(1E)} - 2i|\mathbf{k}_{AF}^{(3)}| \frac{(j^2-2)\mathcal{C}_{(j+1)m}\omega}{j(j+1)} E_{(j+1)m}^{(1E)} - 2i|\mathbf{k}_{AF}^{(3)}| \frac{(j-1)\mathcal{C}_{jm}\omega}{p_{j-1}^2 r} \frac{\partial}{\partial r} E_{(j-1)m}^{(1E)} + 2|\mathbf{k}_{AF}^{(3)}| \frac{\mathcal{C}_{jm}\omega^2}{jp_{j-1}^2} \mathcal{K}_{(j-1)m}^{(0E)} + 2i|\mathbf{k}_{AF}^{(3)}| \frac{(j+2)\mathcal{C}_{(j+1)m}\omega}{p_{j+1}^2 r} \frac{\partial}{\partial r} E_{(j+1)m}^{(1E)} - 2|\mathbf{k}_{AF}^{(3)}| \frac{\mathcal{C}_{(j+1)m}\omega^2}{(j+1)p_{j+1}^2} \mathcal{K}_{(j+1)m}^{(0E)}, \quad (22)$$

$$0 = \mathcal{K}_{jm}^{(0E)} - 2i(k_{AF}^{(3)})_0 \frac{j(j+1)}{\omega r} E_{jm}^{(1B)} - 2i|\mathbf{k}_{AF}^{(3)}| m E_{jm}^{(1E)} - 2|\mathbf{k}_{AF}^{(3)}| (j-1)\mathcal{C}_{jm} E_{(j-1)m}^{(1B)} + 2|\mathbf{k}_{AF}^{(3)}| (j+2)\mathcal{C}_{(j+1)m} E_{(j+1)m}^{(1B)}, \quad (23)$$

where we define $p_j^2 = \omega(\omega+i\sigma) - j(j+1)/r^2$. Note that we could use Eq. (23) to eliminate $\mathcal{K}_{jm}^{(0E)}$. However, for simplicity, we treat $\mathcal{K}_{jm}^{(0E)}$ as a dynamical field on equal footing with $E_{jm}^{(1E)}$ and $E_{jm}^{(1B)}$. The field components $E_{jm}^{(1E)}$ and $E_{jm}^{(1B)}$ correspond to the transverse part of the electric field, which vanishes at the surfaces of a perfect conductor. This implies $\mathcal{K}_{jm}^{(0E)}$ vanishes on the surfaces as well. So we take $E_{jm}^{(1E)}$, $E_{jm}^{(1B)}$, and $\mathcal{K}_{jm}^{(0E)}$ as independent fields that vanish at the boundaries of the cavity.

B. Numerical frequencies

We calculate the resonant frequencies that result from the modified electrodynamics by considering discrete radii $r_n = R + \delta r(n + \frac{1}{2})$, where n is an integer. Defining discrete field coefficients at these points, $E_{njm}^{(1E)}$, $E_{njm}^{(1B)}$, and $\mathcal{K}_{njm}^{(0E)}$, and using discrete derivatives, wave equations (21)-(23) can be written in the form of an infinite-dimensional matrix equation. Discrete resonances cor-

respond to frequencies where nontrivial field configurations exist. These can be estimated by truncating the matrix at finite index values and searching for ω where the truncated matrix is singular. These ω are complex in general. The real parts give the resonant frequencies $\nu = \text{Re}\omega/(2\pi)$, while the ratios of the real and imaginary parts determine the quality factors $Q = -\text{Re}\omega/\text{Im}\omega/2$ of the modes.

In the event that $(k_{AF}^{(3)})_\kappa = 0$, Eqs. (21)-(23) reduce to two wave equations, one for B modes and one for the Schumann E modes. However, nonzero $(k_{AF}^{(3)})_\kappa$ coefficients mix the two parities, and resonances will no longer possess definite E or B parity. We also note that no mixing of fields with different m values occurs, but mixing across j values results from the vector $\mathbf{k}_{AF}^{(3)}$, as expected. As a result, all resonances have definite m values, and m may be fixed in the calculation.

To determine the resonances, we take 100 different r values, $n = 0, \dots, 99$, uniformly spaced between R and $1.01R$. This corresponds to a penetration of about $0.001R \simeq 6.4$ km into the highly conductive ionosphere. For fixed m , we create a matrix including terms corresponding to these n values and the ten lowest j values

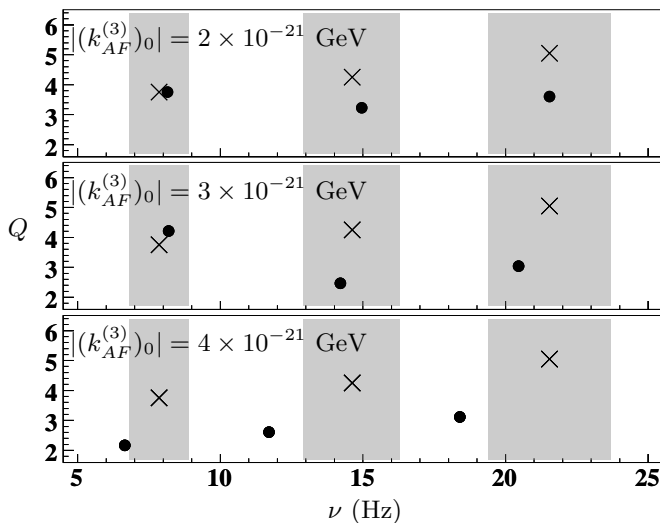


FIG. 1: Calculated resonant frequencies and quality factors for three different values of $|(k_{AF}^{(3)})_0|$. Circles represent the resonances. The Lorentz-invariant limit is shown with \times symbols for comparison. The shaded regions indicate the corresponding widths at half maximum for the three lowest resonances in the Lorentz-invariant case.

that are relevant, $j \geq |m|$. The result is a square matrix with dimension $100 \times 10 \times 3$. We use a row-reduction method to determine its determinant for different values of ω and search for roots.

IV. RESULTS

While any combination of coefficients for Lorentz violation is possible, for simplicity, we next consider the effects of $(k_{AF}^{(3)})_0$ and $|\mathbf{k}_{AF}^{(3)}|$ separately. We first consider a nonzero $(k_{AF}^{(3)})_0$ coefficient. Figure 1 shows the three lowest-frequency resonances for three values of $|(k_{AF}^{(3)})_0|$. The three resonances shown correspond to $j = 1, 2, 3$ and are degenerate in m . The results are independent of the sign of $(k_{AF}^{(3)})_0$ due to the symmetry of this case. The figure also shows the calculated resonant frequency and Q factor for the Lorentz-invariant limit. These are in good agreement with the observed resonances of $(\nu, Q) = (7.8 \text{ Hz}, 4.0)$, $(14.1 \text{ Hz}, 4.5)$, $(20.3 \text{ Hz}, 5.0)$ [6].

From the figure we see that values of $(k_{AF}^{(3)})_0$ on the order of 10^{-21} GeV significantly affect both the resonant frequencies and Q factors. In particular, values of 4×10^{-21} GeV drastically alter all three modes. We therefore adopt a conservative limit of $|(k_{AF}^{(3)})_T| < 4 \times 10^{-21}$ on the time-like part of $(k_{AF}^{(3)})_\kappa$ in the standard Sun-center frame described in Ref. [25]. This translates to a bound of

$$|k_{(V)00}^{(3)}| < 14 \times 10^{-21} \quad (24)$$

on the spherical coefficient from Ref. [23].

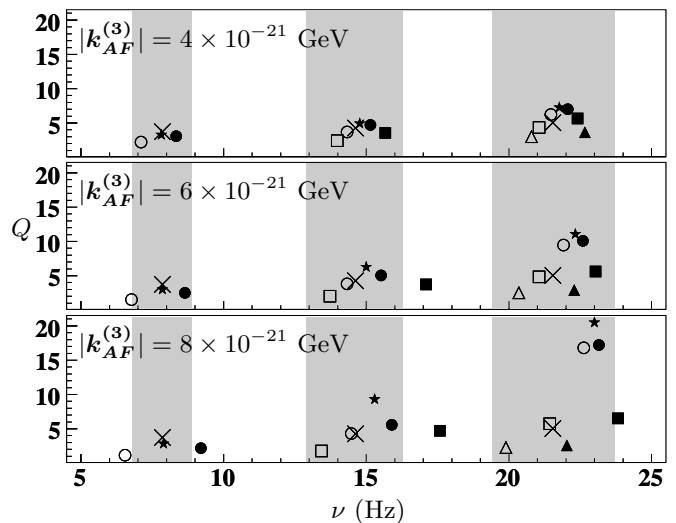


FIG. 2: Resonant frequencies and Q factors for three values of $|\mathbf{k}_{AF}^{(3)}|$. The $|\mathbf{k}_{AF}^{(3)}| = 0$ case (\times symbols) is shown for comparison. The shaded regions indicate the corresponding widths at half maximum for the three lowest resonances in the Lorentz-invariant case. Each plot includes resonances for index values $m = 0$ (stars), $m = \pm 1$ (filled/empty circles), $m = \pm 2$ (filled/empty squares), and $m = \pm 3$ (filled/empty triangles).

Note that the above bound is about two orders of magnitude larger than the naive prediction. This can be understood from the fact that the $(k_{AF}^{(3)})_\kappa$ coefficients mix E - and B -parity modes. The B -parity resonances have frequencies that are at much higher frequencies. This leads to a seesaw effect in the Maxwell equations that suppresses perturbations in the resonances. As a result, relatively large mixing in the wave equations must be present for significant changes to manifest in the low-frequency modes.

Small changes to the conductivity profile can lead to large changes in the resonances, implying that our confidence in the bound on $(k_{AF}^{(3)})_T$ is somewhat weakened by our knowledge of the atmospheric conductivity, which is a complicated and variable system. Much cleaner bounds can be placed on the pseudovector part $\mathbf{k}_{AF}^{(3)}$ since it leads to a breakdown of the usual $2j + 1$ degeneracy among modes with identical j eigenvalues. These bounds too are complicated by imperfections in our conductivity model, including the missing day-side/night-side and polar asymmetries that are present in the real atmosphere [5]. These conventional anisotropies can also break the degeneracies, but they would only add to the effects we are bounding. We therefore neglect them in our analysis. However, these features could be significant in more detailed studies involving field configurations. For example, local fields may experience variability that includes daily and annual fluctuations from conventional physics. Extracting a sidereal dependence caused by the rotation of the Earth with respect to the fixed $\mathbf{k}_{AF}^{(3)}$ vector might

be possible but is beyond the scope of this work.

Figure 2 shows the resonances for three different values of $|\mathbf{k}_{AF}^{(3)}|$. We notice shifts in the frequencies and Q factors as well as the expected $2j+1$ splitting of the resonances. In particular, values of $|\mathbf{k}_{AF}^{(3)}| = 8 \times 10^{-21}$ GeV lead to new resonances separated by frequency intervals comparable to the resonance widths. These multiple resonances would be evident in the data if they existed. Therefore, we take $|\mathbf{k}_{AF}^{(3)}| < 8 \times 10^{-21}$ GeV as a conservative bound. The relation to the spherical coefficients is given by $|\mathbf{k}_{AF}^{(3)}| = \frac{1}{\sqrt{4\pi}}(6|k_{(V)11}^{(3)}|^2 + 3|k_{(V)10}^{(3)}|^2)^{1/2}$ [24]. So our bound leads to two constraints on spherical coefficients for Lorentz violation of

$$\begin{aligned} |k_{(V)11}^{(3)}| &< 12 \times 10^{-21} \text{ GeV}, \\ |k_{(V)10}^{(3)}| &< 16 \times 10^{-21} \text{ GeV}. \end{aligned} \quad (25)$$

These completely bound the vector ($j=1$) dimension-three Lorentz-violating operators. Again, these constraints are less stringent than the naive estimate.

V. DISCUSSION

In this work, we used Schumann resonances in the Earth-ionosphere cavity to place bounds on the order of 10^{-20} GeV on CPT -odd $j=1$ coefficients of the minimal SME. Similar bounds are placed on $j=0$ scalar coefficients assuming the actual conductivity profile of the atmosphere is not significantly different from our model profile. These bounds constitute the first terrestrial bounds on dimension-three Lorentz-violating operators.

While not as sensitive as astrophysical searches for vacuum birefringence, the techniques used in this work test Lorentz invariance in our local neighborhood, giving a bound on coefficients over solar-system length scales. In contrast, astrophysical tests probe Lorentz violation over cosmological scales and may be obfuscated by space-time variations or domains in the Lorentz-violating backgrounds. So local tests play an important role in our search for new physics.

Laboratory-based experiments may also provide local tests of Lorentz invariance. Current cavity experiments utilize high Q factors that allow for sensitivities to $\delta\nu/\nu$ on the order of parts in 10^{15} or better [1]. This suggests improved bounds may be possible in laboratory experiments. A rough estimate yields sensitivities to $(k_{AF}^{(3)})_\kappa$ from about 10^{-25} GeV in optical cavities to around 10^{-30} GeV in lower-frequency microwave cavities.

Future studies involving Schumann resonances may be able to improve on the above bounds. Precise tracking of the resonances may allow for sidereal searches that would indicate rotation violations from $\mathbf{k}_{AF}^{(3)}$. Also, boost violations from $(k_{AF}^{(3)})_\kappa$ coefficients can lead to annual dependences that may be discernible. Regardless, the con-

straints obtained here demonstrate the potential of resonator experiments as tests of dimension-three Lorentz violations in the atmosphere and in the laboratory.

APPENDIX A: SPIN WEIGHT

The problem addressed in this work is most naturally solved in spherical coordinates. Here we use a helicity-based system and covariant-angular-momentum operators. In this appendix, we summarize some of the key identities used in the calculation of the modified wave equations. A fuller discussion of these methods will appear elsewhere [39]. The technique is based on a decomposition into total angular momentum \mathbf{J} and helicity. A type of tensor spherical harmonics called spin-weighted harmonics ${}_s Y_{jm}$ [40] provide orthonormal sets of eigenfunctions of these operators. The index s labels the spin weight, which up to a sign is equivalent to helicity.

The method starts by defining helicity basis vectors, $\hat{\mathbf{e}}_\pm = \hat{\mathbf{e}}^\mp = \frac{1}{\sqrt{2}}(\hat{\mathbf{e}}_\theta \pm i\hat{\mathbf{e}}_\phi)$, $\hat{\mathbf{e}}_r = \hat{\mathbf{e}}^r$, where $\hat{\mathbf{e}}_r = \mathbf{x}/|\mathbf{x}|$ is the radial unit vector, and $\hat{\mathbf{e}}_\theta$ and $\hat{\mathbf{e}}_\phi$ are unit vectors associated with the usual coordinate angles θ and ϕ . The helicity operator $J_r = \hat{\mathbf{e}}_r \cdot \mathbf{J}$ generates local rotations about the radial direction. In the helicity basis, components of 3-dimensional tensors have definite spin weight, which can be determined by counting the number of $+$ and $-$ index values. For example, consider a tensor component $T_{++} = T^{--} = \hat{\mathbf{e}}_+ \cdot \mathbf{T} \cdot \hat{\mathbf{e}}_+$. It is a spin-weight-2 function and can be expanded into the complete set of spin-weight-2 spherical harmonics ${}_2 Y_{jm}$. Another example is the tensor component $T_{-r} = \hat{\mathbf{e}}_- \cdot \mathbf{T} \cdot \hat{\mathbf{e}}_r$, which has a spin weight of -1 . In the present context, the electric and magnetic fields have spin-weight-0 components $E_r = \hat{\mathbf{e}}_r \cdot \mathbf{E}$, $B_r = \hat{\mathbf{e}}_r \cdot \mathbf{B}$ and components $E_\pm = \hat{\mathbf{e}}_\pm \cdot \mathbf{E}$, $B_\pm = \hat{\mathbf{e}}_\pm \cdot \mathbf{B}$, which have a spin weight of ± 1 .

In general, harmonics of a given weight satisfy completeness and orthogonality relations,

$$\sum_{jm} {}_s Y_{jm}^*(\Omega) {}_s Y_{jm}(\Omega') = \delta(\Omega - \Omega'), \quad (A1)$$

$$\int {}_s Y_{jm}^*(\Omega) {}_s Y_{j'm'}(\Omega) d\Omega = \delta_{jj'} \delta_{mm'}. \quad (A2)$$

More generally, they obey

$$\begin{aligned} {}_{s_1} Y_{j_1 m_1} {}_{s_2} Y_{j_2 m_2} &= \sum_{s_3 j_3 m_3} \sqrt{\frac{(2j_1+1)(2j_2+1)}{4\pi(2j_3+1)}} \\ &\times \langle j_1 j_2 (-s_1) (-s_2) | j_3 (-s_3) \rangle \\ &\times \langle j_1 j_2 m_1 m_2 | j_3 m_3 \rangle {}_{s_3} Y_{j_3 m_3}, \end{aligned} \quad (A3)$$

where $\langle j_1 j_2 m_1 m_2 | j_3 m_3 \rangle$ are Clebsch-Gordan coefficients. Note that orthonormality and completion do not extend across harmonics of different spin weight.

The spin-weighted harmonics can be generated for the familiar spin-weight-zero harmonics ${}_0 Y_{jm} = Y_{jm}$ using covariant-angular-momentum operators $J_a =$

$-i\epsilon_{abc}x^b\nabla^c + S_a$, where ∇^a represent covariant derivatives and S_a are covariant spin operators. The spin operators act on tensors in a manner similar to a connection. For example, operating on a tensor $T^b{}_c$, we get $S_a T^b{}_c = S^b{}_{ad} T^d{}_c - S^d{}_{ac} T^b{}_d$, where $S^c{}_{ab} = i\epsilon^c{}_{ab}$. The totally antisymmetric tensor has nonzero components $\epsilon_{+r-} = -\epsilon^{+r-} = i$ in the helicity basis. In general, the components $J_{\pm} = \hat{e}_{\pm} \cdot \mathbf{J}$ raise/lower the spin weight of a function. Specifically acting on the spherical harmonics, we get the ladder relations

$$J_{\pm} {}_s Y_{jm} = -\sqrt{\frac{1}{2}(j(j+1) - s(s \pm 1))} {}_{s \pm 1} Y_{jm} . \quad (\text{A4})$$

Successive operations generate arbitrary ${}_s Y_{jm}$ in terms

of ${}_0 Y_{jm}$.

Finally, there is a useful relationship between the covariant derivatives and the angular-momentum operators in the helicity basis:

$$\begin{aligned} \nabla_r &= \partial/\partial r , \\ \nabla_{\pm} &= \pm(J_{\pm} - S_{\pm})/r . \end{aligned} \quad (\text{A5})$$

These identities help in the reduction of differential tensor equations into radial and angular parts. For example, the divergence of vector V^a becomes $\nabla_a V^a = (\frac{\partial}{\partial r} + \frac{2}{r})V_r + \frac{1}{r}(J_+ V_- - J_- V_+)$, using the properties of the spin operator.

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