regard the average of the consistent uranium-238/lead-206 ages (447 ± 10 m.y.) as the most reliable for the Middle Ordovician (Stones River and Bays formation) time in Tennessee and Alabama, for both experimental and stratigraphical reasons.

We are of the opinion that only after many measurements have been conducted on samples from the various geological periods and have proved to be consistent will it be prudent to try to devise a new geological age-scale.

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OBSERVATIONS OF EARTH–IONOSPHERE CAVITY RESONANCES

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IN 1952, Schumann[1,2] published a detailed description of the resonant modes to be expected for the concentric spherical cavity bounded by the Earth and the lower region of the ionosphere. For such a cavity with perfectly conducting walls, the resonant frequencies should obey the relation:

$$f_n = f_1\sqrt{\left\{\frac{n(n+1)}{2}\right\}} \qquad (1)$$

with a fundamental-mode frequency $f_1 = 10{\cdot}6$ c./s. to be expected for a cavity the size of the Earth.

The ionosphere is, however, a rather poor conductor, and it is furthermore quite difficult to estimate the losses to be expected for oscillations of these extremely low frequencies, because of the tremendously varying ionospheric conditions of electron density and collision frequency in the regions penetrated by the field[3]. These losses result in a relatively low value of Q (the ratio of resonant frequency to frequency separation of the half-power points) for the mode, and the question has been raised as to whether it is indeed so low that it would be difficult to measure the spectrum at all. Another effect of the low value of Q is a shift in the resonant frequency. A general result in the theory of resonant cavities[4] gives:

$$f = f_0\sqrt{\left(1 - \frac{1}{Q}\right)} \qquad (2)$$

as an approximation to the peak frequency f of a mode which would have a resonant frequency f_0 in a perfectly conducting cavity. This expression would no longer be accurate for the very low values of Q to be expected in the Earth–ionosphere cavity, but should serve to indicate that the actual peak frequency of the fundamental mode may be considerably lower than 10·6 c./s.

It may be assumed that the cavity is continually being excited by lightning strokes all over the world. To the extent that radio noise at these frequencies is a result of such excitation, the power spectrum of the noise should show peaks with frequencies and values of Q corresponding to the various resonant modes. There has been some indication that variations in the vertical electric field below about 20 c./s. are largely local in origin[5] (for example, moving clouds, pollution of air by combustion products), which would further inhibit the observation of cavity effects.

An earlier attempt[6] to observe cavity effects of modes of higher order with frequencies greater than 60 c./s. produced some suggestive results, but they were not statistically significant. It was therefore decided, despite the problems just mentioned, to attempt to observe the cavity effects on the spectrum of radio noise at and just above the fundamental frequency.

As in the previous attempt[6], it was planned to produce the final results by simulated spectrum analysis on a digital computer. For this and other reasons, it was decided to record the raw data digitally in such a format that the original tape recorded at the field site could be read directly into the Lincoln Laboratory IBM 709 computer for processing.

The equipment used to record the radio noise was assembled at a field site in Ipswich, Mass. A block diagram is shown in Fig. 1. The (receiving) antenna was a 120-ft. metal tower which, of course, responded to variations in the vertical electric field. The signal was passed through a low-pass filter which cut off at about 75 kc./s. to prevent strong radio-frequency fields from overloading succeeding stages. A cathode follower was then used to reproduce the signal at a lower impedance-level. This was followed by an audio filter which cut off at about 35 c./s. and was designed to have a minimum at 60 c./s. to minimize interference by power lines (none was ever observed following this filter). The filtered signal was then fed into an audio amplifier. All this equipment was powered by batteries and was located directly at the base of the antenna. The frequency response of the system, measured by replacing the antenna with a signal generator of like impedance, is shown in Fig. 2.

The signal was conducted into the building which contained the recording equipment through an insulated shielded cable. In order to eliminate ground return currents which would result from the small potential difference between the ground at the antenna and the ground at the building, an amplifier with unity

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* Operated with support from the U.S. Army, Navy and Air Force.

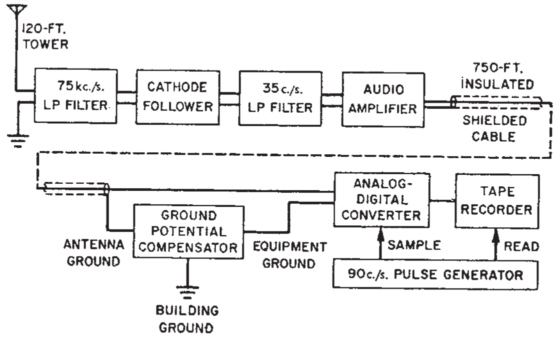

Fig. 1. Recording equipment

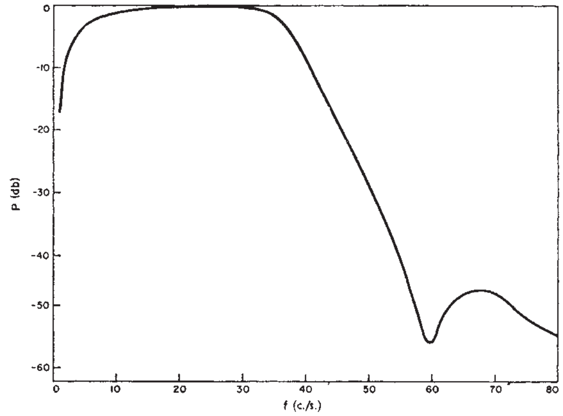

Fig. 2. Frequency response of recording equipment

gain was used to sense this potential difference and maintain the recording equipment ground at the potential of the antenna ground. Finally, the signal was sampled 90 times per sec. and the voltage was then converted to a six-bit number, that is, one of 64 levels, and recorded on IBM magnetic tape. Since the highest effective frequency in the signal was less than 45 c./s., no information is lost in the sampling process except for the small quantization error.

The day which was chosen for making these observations, June 27–28, 1960, appeared to be most favourable for minimizing undesired effects. The sky was absolutely clear, the humidity was low (for New England), and the air was fairly calm (straight, sharp jet-vapour trails indicated that this was also true at higher levels). Atmospheric interference in the high-frequency band seemed quite low for the summer-time.

Recordings of the extremely low-frequency radio noise of duration slightly longer than 12 min. were taken on an average once every 2 hr. around the clock for the day indicated. Also recorded for test purposes were sine waves of various frequencies in the pass-band, produced by an oscillator which was substituted on those occasions for the antenna.

During parts of the recording periods the analogue signal that was presented to the digital apparatus was also recorded on paper tape, and it was always monitored on an oscilloscope. A typical sample of the recorded noise is given in Fig. 3. The only exceptions to this apparently random wave-form were large impulses (which corresponded to nearby lightning disturbances heard on high-frequency) which saturated the pen recorder and the digital recorder for periods of up to a second. These increased in frequency of occurrence and intensity during the night and diminished again the next day, although not quite to the level of the previous day. Apart from these bursts, no wave-forms different in character from that shown in Fig. 3 were ever seen. (This contrasts with findings of Schumann and König[7,8], who have reported observing signals of sinusoidal character with frequencies near 8 or 9 c./s. on similar paper recordings. Some differences in the shape of their pass-band (principally due to a notch filter they used to eliminate 16$\frac{2}{3}$ c./s. interference) may help explain the disparity in observed wave-forms.)

The filtered radio noise signal was compared on an oscilloscope with a calibrated 10 c./s. oscillator, and the root mean square noise voltage was estimated to be about 20 mV.

A programme was written for the IBM 709 computer (largely adapted from the one written for the earlier attempt[6]), which read in a 12-min. block of noise signal from the original tape and, in effect, passed it through a 1 c./s. filter, then squared and integrated the output to give the power in that 1 c./s. band. This was done for all integral frequencies between 5 and 34 c./s. and the results were multiplied by a correction factor to compensate for the system response of Fig. 2.

In mathematical terms, the computer simulated a narrow-band filter the impulse response of which is given by :

$$h(f,t) = \begin{cases} \sin \omega t & 0 < t < 1 \\ 0 & \text{elsewhere} \end{cases} \qquad (3)$$

where $\omega = 2\pi f$ and f is an integral frequency between 5 and 34 c./s. The frequency response of this filter is given by :

$$\left| H(f') \right|^2 = \frac{\sin^2 \pi (f' - f)}{\pi^2 (f' - f)^2} \qquad (4)$$

and is shown in Fig. 4 as the solid curve. The peak at the centre would be at the frequency f, and the nulls occur at all other integral frequencies. If we

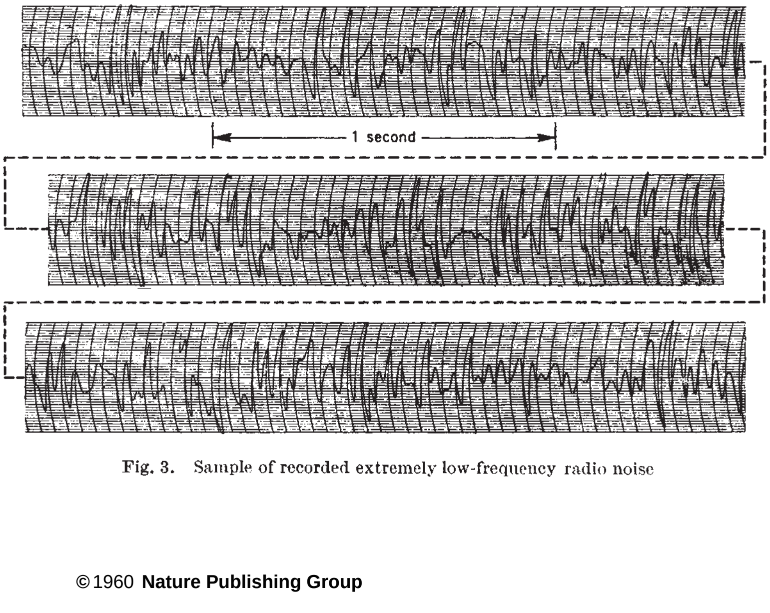

Fig. 3. Sample of recorded extremely low-frequency radio noise

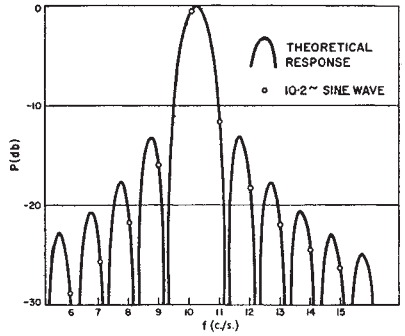

Fig. 4. Theoretical frequency response (solid curve) of 1 c./s. filter used in processing. Points are results of processing a sine wave of about 10·2 c./s.

call the input signal $x(t)$, then the output of this filter is:

$$y(f,t) = \int_{t-1}^{t} x(\tau) \sin \omega(t - \tau) d\tau \qquad (5)$$

$$= \left(\int_{t-1}^{t} x(\tau) \cos \omega\tau d\tau\right) \sin \omega t - \left(\int_{t-1}^{t} x(\tau) \sin \omega\tau d\tau\right) \cos \omega t$$

The power in this band is then estimated from:

$$P(f) = \int_{0}^{T} y^2(f,t) dt \qquad (6)$$

where $T = 720$ sec. The computer performs all of the operations indicated on the input tape, substituting sums over the sample points for integrals.

The integration process just outlined reduces the fluctuations in the estimate of power $P(f)$, which is a random variable derived from the sample function $x(t)$, so that smaller differences in mean power may be discerned. By a calculation analogous to that presented in ref. 6, it can be shown that the standard deviation $\sigma(P)$ is given by:

$$\frac{\sigma(P)}{E(P)} = \frac{2}{\sqrt{3TW}} \qquad (7)$$

where $E(P)$ (the quantity we seek) is the mean value of P, W is the 1 c./s. band-width of the filter, and T is the integration time of 720 sec. (This equation was erroneously given as being smaller by a factor $\sqrt{2}$ in ref. 6.) It can therefore be expected that random fluctuations slightly larger than 4 per cent of the mean power (± 0·18 db.) will appear in the computed spectra, and considerably larger deviations may be accepted as significant spectral structure.

In order to test the system, a sine-wave signal of about 10·2 c./s. was recorded and processed. The results are the points shown in Fig. 4. (More accurately, each point is the result of centring the filter on the frequency indicated below it and finding the response at 10·2 c./s. Due, however, to the symmetry between f and f' in equation (4), the points should all lie on the response curve centred at 10·2 c./s.) It can be seen that the system operates as expected. A further result of these tests indicates that the recorded radio noise is nearly 30 db. above the system noise, so that the latter should not be a consideration in the results.

Ten records of noise were obtained which could be properly processed. Two representative spectra resulting from this processing are shown in Fig. 5. The power, as stated earlier, is computed only at integral frequencies. These points are joined by straight-line segments for ease in reading them, not to suggest the actual shape of the spectrum.) The records were taken around 4.20 p.m. E.S.T. and 3.50 a.m. E.S.T. The mode structure of these spectra is immediately obvious (as it is on all the remaining records processed), with the power differences being much larger than the predicted statistical fluctuations. The only systematic difference that could be observed among the records is the behaviour below the lowest mode frequency. As already observed, the night-time records were characterized by more frequent and stronger lightning bursts. These would saturate the recorder for periods of a fraction of a second to a second, and would naturally appear as strong enhancements of the spectrum in the region of a few cycles. The very large enhancement observed in Fig. 5 (broken line) at 5 c./s. (and undoubtedly down to about 1 c./s.) is representative of the regular increase and subsequent decrease in close correspondence to the previously noted lightning activity.

Fig. 6 is a composite of all ten processed records. It therefore represents two hours of integration, although the individual 12-min. periods are taken from all parts of the day rather than continuously. The random fluctuations are smoothed out even further, and in this case the points are joined by a smooth curve that is an attempt to suggest the probable shape of the spectrum. No attempt is made to follow the points below the dominant mode where the behaviour is systematically variable, as discussed in the preceding paragraph, and where the sum therefore does not constitute further smoothing.

The resultant spectrum is smooth enough to be read to ± 0·1 c./s. on the original curve, and the peak frequencies were determined for each of the five modes in the range. These are listed in Table 1. Estimates can also be made of the Q of the cavity for each of the modes. The fundamental mode has $Q = 4$, and the value increases regularly (although it is difficult to obtain accurate estimates) until for the fifth mode, $Q \approx 6$.

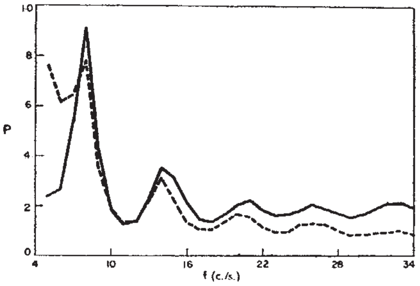

Fig. 5. Typical spectra of recorded noise (scale of P is linear). Full line, day-time record; broken line, night-time record

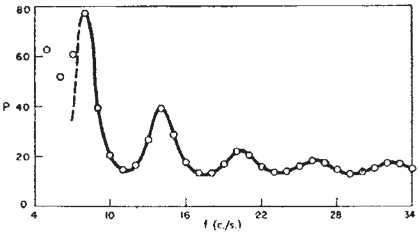

Fig. 6. Composite spectrum of ten 12-min. records of extremely low-frequency noise (scale of P is linear)

Table 1

n	f_n (c./s.)	$\sqrt{\left\{\frac{2}{n(n+1)}\right\}} \times f_n$ (c./s.)
1	7·8	7·8
2	14·1	8·14
3	20·3	8·29
4	26·4	8·35
5	32·5	8·39

Note added in proof. The frequency 7·8 c./s. for the first peak was actually obtained from a more detailed examination of the results, carried out after the submission of this article, in which the simulated filters had a bandwidth of 1/3 c./s. and were separated in frequency by that amount. The points in Fig. 6 are insufficient to determine the first peak to within 0·1 c./s., and an earlier estimate of 8·0 c./s. was obtained from some of the individual records relatively free of corrupting impulse noise at the lower frequencies.

Except for the low value of f_1, the results seem to be quite consistent with the predicted behaviour. The frequency progression is about what is expected from equation (1). The quantity $\sqrt{\left\{\frac{2}{n(n+1)}\right\}} \times f_n$, which from (1) should be f_1, is also given in Table 1. As can be seen, these values are close to the fundamental frequency but tend systematically to increase with n. This is just the behaviour to be expected, since Q is also increasing with n and the values f_n are therefore decreased proportionately less at the higher frequencies.

It is of interest to recall a marginal observation made in the earlier work[6] but rejected because of insufficient supporting evidence. There seemed to be some indication there of modes at frequencies of about 63, 69, 75, 81 and 87 c./s. with $Q \approx 10$. These values extrapolated back to a fundamental frequency of about 8·5 c./s. In the light of the present (stronger) evidence, it seems quite probable that the earlier result did in fact constitute an observation of the continuation of the series of modes reported here.

An incidental result may be noted in connexion with the remarks on the nature of radio noise below 20 c./s. From the depth of the valley below the lowest mode on some of the records, for example, Fig. 5 (full line), it seems likely that, at least under the conditions of these measurements, the noise due to local charge fluctuations in the air does not become dominant until below 5 c./s.

In conclusion, it should be mentioned that while these observations seem to confirm the stable, round-the-clock structure of the Earth-ionosphere cavity modes, they were, after all, made all in one day at one place. Since the properties of the modes (peak frequency and Q) depend on the nature of the lower ionosphere, it is easily conceivable that there are effects that were missed in these observations. There is certainly room for further work, both experimental and theoretical, to obtain a better understanding of this phenomenon.

We thank J. V. Harrington and W. B. Davenport, jun., for suggesting this problem. We are also grateful to R. Cherry, who carried out the machine computations, to R. E. Gay, who implemented the analogue-to-digital conversion equipment, and to J. W. Craig, who designed the audio filter.

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THEORETICAL LIMITING THICKNESSES FOR SINGLE SCATTERING IN ELECTRON MICROSCOPY

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THE quantitative aspects of electron microscopy have been examined recently by a number of authors[1-7] and it is generally agreed that accurate measurements of mass-thickness, that is, the product of thickness and density of the specimen, can be made only on specimens for which the contrast in the electron-microscope image is proportional to the mass-thickness (w). (Contrast is here defined as $\ln(I_0/I)$, where I_0 and I are respectively the intensities of the electron beam incident on the specimen and that received at the image plane.) The maximum limit ($w_{\text{lim.}}$) of the range within which $\ln(I_0/I)$ is proportional to w is therefore of importance, since its value for given operating conditions of the electron microscope governs the choice of specimens suitable for quantitative work.

Values of $w_{\text{lim.}}$ have been deduced on theoretical grounds, based essentially on the point of transition from single to multiple electron scattering as the mass-thickness of a specimen increases. Experimental results[1,8-11] produced by measurements on the variation of image-contrast with w show that for a variety of operating conditions values of $\ln(I_0/I)$ up to 3 and values of w in the range 0–100 μgm. cm.$^{-2}$ are within the range where the contrast is directly proportional to the mass-thickness of the specimen.

Theoretically the situation is rather confused. At the present time, two theories of electron scattering are in current use, due to Leisegang[12] and Lenz[13] respectively. Leisegang's theory is restricted to a consideration of elastic scattering only, while Lenz treats both elastic and inelastic processes. The two elastic scattering theories are fundamentally the same, since they are based on the same quantum-mechanical treatment (for example, Bohm[14]). Using these two theories of scattering, essentially three different theoretical definitions of $w_{\text{lim.}}$ have been proposed, giving values of the limiting thickness which differ considerably from one another both in magnitude and in physical implication.