

**Tau/QED-07/09**  
**BAD 2222, version 6**

Collaboration-Wide Review  
11 July 2009 to 22 July 2009

<b>Primary BAD</b>	2222, version 6  Analysis of the tau->KsPinu mass spectrum, and measurement of tau->KsPinu and tau->KsPiPi0nu branching fractions.
<b>Author list</b>	Banerjee, Swagato; Cowan, Glen; Paramesvaran, Sudarshan
<b>Review Committee</b>	comm392, members: Barlow, Roger; Nugent, Ian M.; Sekula, Stephen J. (chair)
<b>Target</b>	
<b>Result type</b>	
<b>Supporting BAD(s)</b>	BAD #2055 Analysis of the tau-> Ks Pi nu invariant mass spectrum using the BaBar detector
<b>Changes since preliminary result</b>	This result performs a fit to the mass spectrum of the Ks pi system and measures the K* mass and width, as well as looking for evidence of other resonance contributions to the spectrum.
<b>BAIS/CWR Comments</b>	This resulted is aimed for physics sign-off in time for presentation at DPF and any other following summer conference. A separate and full publication of these results will follow this preliminary result of the fits to the mass spectrum.
<b>Institutional Reading Groups</b>	2b. Budker, UCLA, Colorado, Royal Holloway, McGill, MIT, Trieste

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# Analysis of the decays $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$ and $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$ using the *BABAR* detector

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The *BABAR* Collaboration

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July 11, 2009

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## Abstract

8 We present studies of the decays  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  and  $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$  using  $384.6 \text{ fb}^{-1}$  of  $e^+e^-$   
 9 collision data provided by the PEP-II accelerator, operating primarily at  $\sqrt{s} = 10.58 \text{ GeV}$ , and  
 10 recorded using the *BABAR* detector. For  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  we carry out a fit to the hadronic mass  
 11 distribution which yields values for the  $K^*(892)$  mass and width:

$$M(K^*(892)^-) = 894.30 \pm 0.19 \text{ (stat.)} \pm 0.19 \text{ (syst.) MeV ,}$$

$$\Gamma(K^*(892)^-) = 45.56 \pm 0.43 \text{ (stat.)} \pm 0.57 \text{ (syst.) MeV .}$$

12 We analyse the possibility of other resonances being present in this mass spectrum, and conclude  
 13 that a combination of  $K^*(800)$ ,  $K^*(892)$  and  $K^*(1410)$  provides a good description of the data. A  
 14 fit without the  $K^*(800)$ , i.e., using only a  $K^*(892) + K^*(1410)$  combination, can also provide a good  
 15 description of the data if the modeling of backgrounds were to be incorrect by a significant amount.  
 16 Studies are presented which consider how the principal background mode must be altered for a two-  
 17 resonance model to fit the data. The altered background model is reasonably consistent, within  
 18 experimental uncertainty, to the data used for the Monte Carlo generation of this background. It  
 19 is therefore not possible at this moment to confirm the necessity of a  $K^*(800)$  resonance to fit the  
 20  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  invariant mass spectrum.

21 In addition we have studied the hadronic mass distribution for the decay  $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$ . The  
 22 measurement is used to improve the Monte Carlo simulation of this decay, which was needed for a  
 23 background estimate in the analysis of the  $K_S^0 \pi^- \nu_\tau$  mode. The branching ratio for  $\tau^- \rightarrow K^0 \pi^- \pi^0 \nu_\tau$   
 24 is measured to be

$$\mathcal{B}(\tau^- \rightarrow K^0 \pi^- \pi^0 \nu_\tau) = (0.342 \pm 0.006 \text{ (stat.)} \pm 0.015 \text{ (sys.)})\% .$$

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*SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309*

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Work supported in part by Department of Energy contract DE-AC02-76SF00515.

# 1 INTRODUCTION

Studies are presented of the hadronic mass distributions for the decays  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  and  $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$  (throughout the note, charge conjugate modes are included).

Since the 2008 [1] analysis, important improvements have been carried out in the modeling of the background in that mode from  $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$ , which are presented here. The measurements have been used to tune the TAUOLA Monte Carlo model [2] used to describe the  $\tau$ -lepton decay. This is then used to model the background from  $K_S^0 \pi^- \pi^0 \nu_\tau$  in the analysis of  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$ . The resulting branching ratio for  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  is fully consistent with the 2008 preliminary value.

For the decay  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  we carry out a fit of the hadronic mass distribution that yields precise values for the mass and width of the  $K^*(892)$  as well as information on other resonances present in the distribution. The Belle collaboration recently published [3] an analysis of the hadronic mass distribution for this mode, where they quoted measurements of the  $K^*(892)$  mass and width in significant disagreement with the values determined by the PDG [8].

It is known that the  $K^*(892)$  provides the main contribution to the  $K\pi$  mass spectrum, but prior to the Belle measurement there had been no definitive evidence for additional resonances (e.g., scalar or tensor contributions). Although these are expected theoretically, the large data sample required to perform a detailed study had not been available. Belle has proposed the mass distribution should contain a contribution from a scalar meson at 800 MeV, as without it they cannot sufficiently describe the low end of the spectrum.

The event selection for the  $K_S^0 \pi^- \nu_\tau$  mode is described in Ref. [1], where details are also given about the data and Monte Carlo samples used. For the  $K_S^0 \pi^- \pi^0 \nu_\tau$  final state we begin with the same selection criteria but then include an additional  $\pi^0$ , as described in Section 3.1.

The analysis of  $K_S^0 \pi^- \pi^0 \nu_\tau$  is described in Section 4. This includes both the measurement of the branching ratio and the use of the observed mass distribution to improve the TAUOLA Monte Carlo generator.

The analysis of the hadronic mass distribution for the  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  final state is described in Section 5, the fit results are presented in Section 5.5, and the systematic uncertainties are discussed in Section 5.6. The results are summarized and conclusions given in Sections 5.5 and 6.

# 2 THE BABAR DETECTOR AND DATASET

The BABAR detector is described in detail in Ref [4]. Charged particles are detected and their momenta measured with a 5-layer double sided silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH) inside a 1.5 T superconducting solenoidal magnet. A ring-imaging Cherenkov detector (DIRC) is used for the identification of charged particles. Energies of neutral particles are measured by an electromagnetic calorimeter (EMC) composed of 6,580 CsI(Tl) crystals, and the instrumented magnetic flux return (IFR) is used to identify muons.

The analysis described in this paper is based on data taken using the BABAR detector at the PEP-II collider [5] located at the SLAC National Accelerator Laboratory in the data-taking periods between October 1999 and August 2006. During this period a total of  $384.6 \text{ fb}^{-1}$  of data was recorded with a cross-section for  $\tau^+ \tau^-$  pair production of  $(0.919 \pm 0.003) \text{ nb}$  [6]. This data sample contains over 700 million  $\tau$  decays.

Monte Carlo (MC) studies of simulated signal and background events were carried out using various MC samples. The  $\tau$  MC events studied were generated with KK2f [7] and decayed with TAUOLA [2] using  $\tau$  branching fractions based on Ref [8]. In the MC, the  $\tau^-$  decays to  $K_S^0 \pi^-$  via

70 the  $K^*(892)^-$  resonance with a branching fraction of 0.90%. Non- $\tau$  hadronic and dilepton MC  
 71 samples are used for studying the non- $\tau$  backgrounds.

72 Important improvements to TAUOLA's modeling of the decay  $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$  were undertaken  
 73 as part of this analysis, as described in Section 4.

### 74 3 EVENT SELECTION

75 The analysis uses  $384.6 \text{ fb}^{-1}$  of data taken between October 1999 and August 2006 (runs 1 through  
 76 5). The event selection for  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  is described in Ref. [1]. For  $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$  one begins  
 77 with the same selection as for  $K_S^0 \pi^- \nu_\tau$  and then requires an additional  $\pi^0$ , which is described in  
 78 Section 3.1.

#### 79 3.1 EVENT SELECTION FOR $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$

80 Using the selection criteria in the analysis of  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  as a starting point, we then made the  
 81 following adjustments to select events for  $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$ . We require:

- 82 • exactly one identified  $\pi^0$  in the event;
- 83 • the trajectory of the  $\pi^0$  must be within 90 degrees of the  $K_S^0 \pi^-$  momentum vector. This  
 84 ensures that the  $\pi^0$  is more likely to be from the same  $\tau$  as the  $K_S \pi$ ;
- 85 • the neutral energy not attributed to the  $K_S^0$  or the  $\pi^0$  must be less than 100 MeV. This should  
 86 be very small anyway, but the cut is to reject unwanted photons;
- 87 • the energy of the  $\pi^0$  in the center-of-mass system must be greater than 1.2 GeV. This cut is  
 88 to remove the large background contribution in the region below 1.2 GeV.

89 As an example, Fig. 1 shows the distribution of the  $\pi^0$  energy. The combination of all cuts results  
 90 in a signal efficiency of 0.500% with a purity of 93%; more details are given in Section 4.

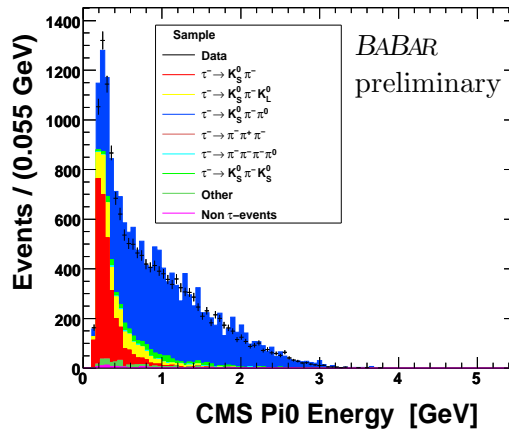


Figure 1: Distribution of the  $\pi^0$  energy.

## 91 4 ANALYSIS OF $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$

92 For the decay mode  $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$  we have measured the branching ratio, which is described  
 93 in Section 4.1, and we also carry out an analysis of the hadronic mass distribution, shown in  
 94 Section 4.2.

### 95 4.1 Branching fraction measurement

96 Defining the signal process to be  $\tau^\pm \rightarrow K_S^0 \pi^\pm \pi^0 \nu_\tau$  with  $K_S^0 \rightarrow \pi^+ \pi^-$ , our overall signal efficiency  
 97 is found to be

$$\varepsilon_{\text{sig}} = \frac{N_{\text{sig}}^{\text{sel}}}{N_{\text{sig}}^{\text{gen}}} = 0.00500 \pm 0.00008 \text{ (stat.)} . \quad (1)$$

98 Redefining the signal process to be  $\tau^- \rightarrow \bar{K}^0 \pi^- \pi^0 \nu_\tau$ , the signal efficiency becomes  $\varepsilon'_{\text{sig}} =$   
 99  $\varepsilon_{\text{sig}} \times \mathcal{B}(\bar{K}^0 = K_S^0) \times \mathcal{B}(K_S^0 \rightarrow \pi^+ \pi^-)$ . The branching fraction  $\mathcal{B}(\tau^- \rightarrow \bar{K}^0 \pi^- \pi^0 \nu_\tau)$  is estimated by

$$\mathcal{B}(\tau^- \rightarrow \bar{K}^0 \pi^- \pi^0 \nu_\tau) = \frac{1}{2N_{\tau\tau}} \frac{N_{\text{data}} - N_{\text{bkg}}}{\varepsilon'_{\text{sig}}}, \quad (2)$$

100 where  $N_{\tau\tau}$  is the total number of  $\tau^+ \tau^-$  pairs in the real data,  $N_{\text{data}}$  is the number of selected events  
 101 in real data,  $N_{\text{bkg}}$  is the number of background events estimated from Monte Carlo.

Table 1:  $\mathcal{B}(\tau^- \rightarrow \bar{K}^0 \pi^- \pi^0 \nu_\tau)$  measured in this analysis.

Sample	$\mathcal{B}(\tau^- \rightarrow \bar{K}^0 \pi^- \pi^0 \nu_\tau)$ [%]
<i>e</i> -tag	$0.353 \pm 0.008 \text{ (stat)} \pm 0.016 \text{ (syst)}$
$\mu$ -tag	$0.329 \pm 0.008 \text{ (stat)} \pm 0.016 \text{ (syst)}$
Combined	$0.342 \pm 0.006 \text{ (stat)} \pm 0.015 \text{ (syst)}$

102 Most of the systematic uncertainties related to this measurement are common to those of the  
 103 original  $\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau$  analysis, details of which are given in Ref. [1].

104 In addition to the uncertainties included from the  $\tau^- \rightarrow \bar{K}^0 \pi^- \nu_\tau$  analysis, a 3% systematic for the  
 105  $\pi^0$  efficiency correction was added in quadrature. A summary of the systematic uncertainties is  
 106 given on the next page. It has been checked that the effect on the estimated branching fraction of  
 107 varying the cut on the  $\pi^0$  energy is small compared to the quoted systematic uncertainty.

### 108 4.2 Hadronic mass distributions for $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$

109 Figure 2 shows the hadronic mass distributions from the  $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$  decays for different  
 110 combinations of the final state hadrons:  $\pi^- \pi^0$ ,  $K_S^0 \pi^-$ ,  $K_S^0 \pi^0$  and  $K_S^0 \pi^- \pi^0$ . The plots show the  
 111 observed distributions (data points), and Monte Carlo predictions for signal (blue) and background.  
 112 The signal predictions are based on the original TAUOLA generator and show large discrepancies with  
 113 the measurements. Figure 2(b) shows specifically how the signal Monte Carlo had no  $K^*(892)$   
 114 resonance, while our data clearly exhibit this peak.

115 The measured distributions were used to improve the form factors in the TAUOLA generator lead-  
 116 ing to a greatly improved description of the  $\tau^- \rightarrow K_S^0 \pi^- \pi^0$  decay. Figure 3 shows the background  
 117 subtracted measurements for the same mass combinations as in Fig. 2 along with both the old and

Table 2: Summary of the systematic uncertainties as they feed into the measurement of  $\mathcal{B}(\tau^- \rightarrow K^0 \pi^- \pi^0 \nu_\tau)$ .

Systematic	$e$ -tag	$\mu$ -tag	Combined
Tracking	0.58%	0.58%	0.58%
$K_S^0$ Efficiency	1.40%	1.40%	1.40%
PID	1.45%	1.68%	1.50%
$\mathcal{L} \times \sigma_{\tau\tau}$	0.70%	0.70%	0.70%
MC signal	1.87%	2.08%	1.39%
MC background	0.28%	0.30%	0.20%
$\tau$ backgrounds	1.37%	1.37%	1.37%
Modelling	0.37%	0.37%	0.37%
$\pi^0$ efficiency	3.30%	3.30%	3.30%
Total	4.55%	4.77%	4.41%

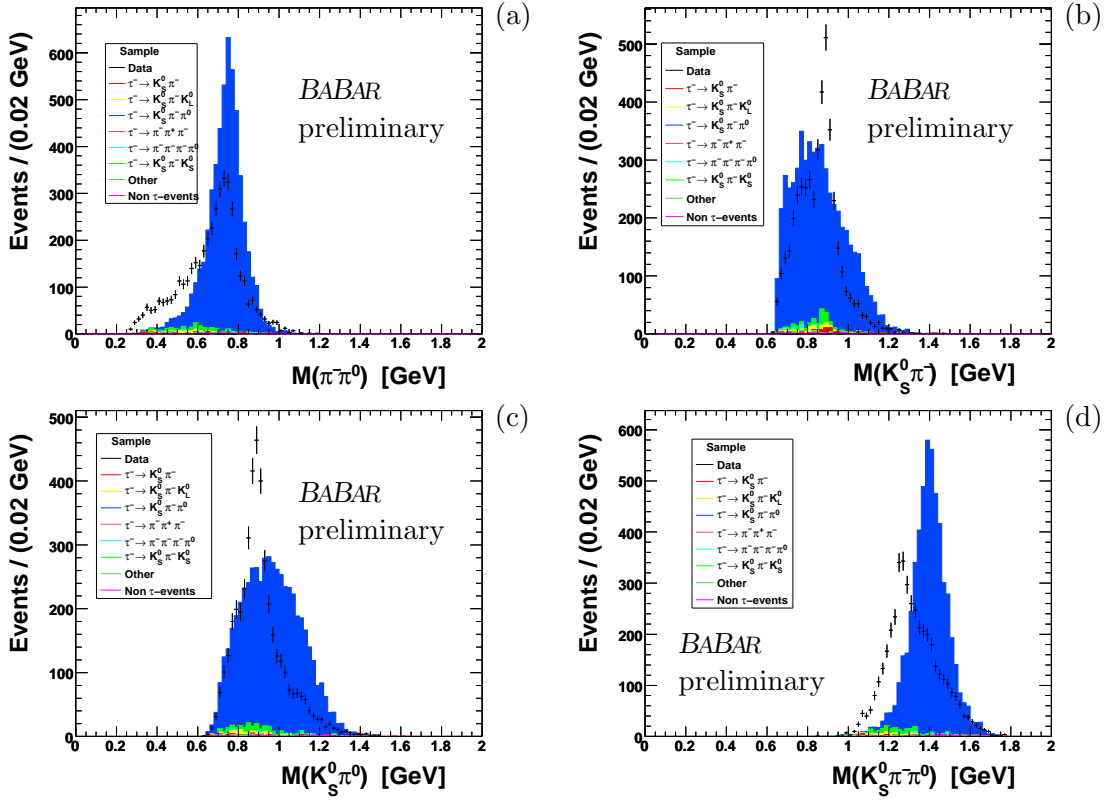


Figure 2: Invariant mass distributions of different hadron combinations from the  $K_S^0 \pi^- \pi^0$  final state showing data (points) and MC predictions for background and signal (in blue) based on the original version of TAUOLA: (a)  $\pi^- \pi^0$ , (b)  $K_S^0 \pi^-$ , (c)  $K_S^0 \pi^0$  and (d)  $K_S^0 \pi^- \pi^0$ . The overall normalization of the MC predictions is scaled to be the same as that of the data.

118 improved Monte Carlo predictions. This new Monte Carlo is used to determine the efficiency for  
 119 our measurement of the  $\tau^- \rightarrow K_S^0 \pi^- \pi^0$  branching ratio described above, and it is also used in our  
 120 analysis of the  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  mass spectrum.

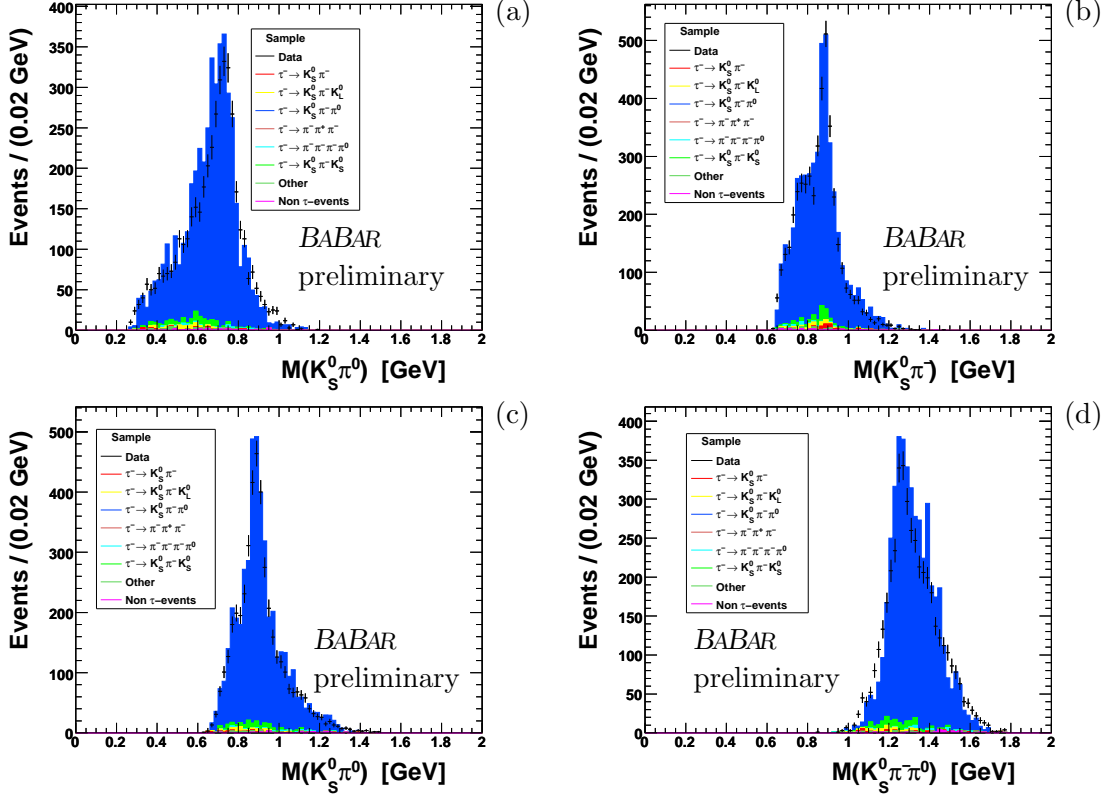


Figure 3: Invariant mass distributions of different hadron combinations from the  $K_S^0 \pi^- \pi^0$  final state showing data (points) MC predictions for background and signal (in blue) based on the tuned version of TAUOLA: (a)  $\pi^- \pi^0$ , (b)  $K_S^0 \pi^-$ , (c)  $K_S^0 \pi^0$  and (d)  $K_S^0 \pi^- \pi^0$ . The overall normalization of the MC predictions is scaled to be the same as that of the data.

## 121 5 ANALYSIS OF $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$

122 The analysis of the decay  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  is a fit of the hadronic mass distribution to a parametric  
 123 function describing the resonant structure. From this we obtain precise values for the mass and  
 124 width of the  $K^*(892)$  as well as information on other resonances present in the spectrum.

### 125 5.1 Fit methodology and signal model

126 We denote the number of events found in bin  $i$  (without background subtraction) by  $n_i$ . The  
 127 prediction for the expectation value of  $n_i$ ,  $\nu_i = E[n_i]$ , can be written

$$\nu_i = \sum_{j=1}^M R_{ij} \mu_j + \beta_i, \quad (3)$$

128 where  $\beta_i$  is the number of background events,  $\mu_j$  is the predicted number of signal events in bin  
 129  $j$  before detector effects (the “true” distribution), and  $R_{ij}$  is a response matrix that reflects the  
 130 limited efficiency and resolution of the detector. The value of  $R_{ij}$  is the probability for a signal  
 131 event created in bin  $j$  to be found in bin  $i$ ,

$$R_{ij} = P(\text{event found in bin } i | \text{event created in bin } j), \quad (4)$$

132 and thus the efficiency for bin  $j$  is found by summing over all bins where the event could be found,  
 133 i.e.,

$$\varepsilon_j = \sum_{i=1}^N R_{ij} = P(\text{event found anywhere} | \text{event created in bin } j). \quad (5)$$

134 The predicted number of events in bin  $j$  of the true distribution can be written

$$\mu_j = \mu_{\text{tot}} \int_{\text{bin } i} f(m; \vec{\theta}) dm, \quad (6)$$

135 where  $m$  denotes the  $K_S^0 \pi^-$  invariant mass and  $\vec{\theta}$  represents a set of parameters. When calculating  
 136 the value of the signal pdf in each bin, a numerical integration should be performed over the bin  
 137 width. Where the bin width is small, however, it is a good approximation to take the pdf’s value  
 138 in the centre of the bin and multiply by the bin width. This is done except in the regions where  
 139 the distribution is varying rapidly, i.e., near the  $K^*(892)$  peak and also just above the kinematic  
 140 threshold.

141 The probability density function (pdf)  $f(m; \vec{\theta})$  can be written

$$f(m; \vec{\theta}) \propto \frac{1}{s} \left(1 - \frac{s}{m_\tau^2}\right) \left(1 + 2 \frac{s}{m_\tau^2}\right) P \left( P^2 |F_V|^2 + \frac{3(m_K^2 - m_\pi^2)^2}{4s(1 + 2 \frac{s}{m_\tau^2})} |F_S|^2 \right). \quad (7)$$

142 where  $s = m^2$ . Here the vector form factor  $F_V$  is given by

$$F_V = \frac{1}{1 + \beta + \gamma + \dots} [BW_{K^1}(s) + \beta BW_{K^2}(s) + \gamma BW_{K^3}(s) + \dots]. \quad (8)$$

143 This form allows for the  $K^*(892)$  and two additional vector resonances. The quantities  $\beta$  and  $\gamma$  are  
 144 complex interference terms between the resonances, and the BW terms refer to the relativistic  
 145 Breit-Wigner functions for the specific resonance, given by

$$BW_R(s) = \frac{M_R^2}{s - M_R^2 + i\sqrt{s}\Gamma_R(s)}. \quad (9)$$

146 The energy dependent width is given by

$$\Gamma_R(s) = \Gamma_0 R \frac{M_R^2}{s} \left( \frac{P(s)}{P(M_R^2)} \right)^{2\ell+1}, \quad (10)$$

147 where



$$P(s) = \frac{1}{2\sqrt{s}} \sqrt{(s - M_+^2)(s - M_-^2)}, \quad (11)$$

148 and where  $M_- = M_K - M_\pi$ ,  $M_+ = M_K + M_\pi$ , and  $\ell$  is orbital angular momentum. Thus one has  
 149  $\ell = 1$  if the  $K\pi$  system is from a P-wave (vector), or  $\ell = 0$  if the  $K\pi$  system is from an S-wave  
 150 (scalar).

151 The scalar form factor requires a different parametric function and can include contributions  
 152 from the  $K^*(800)$  and  $K^*(1430)$  signals. This is

$$F_S = \varkappa \frac{s}{M_{K^*(800)}^2} BW_{K^*(800)}(s) + \lambda \frac{s}{M_{K^*(1430)}^2} BW_{K^*(1430)}(s). \quad (12)$$

153 We have also investigated using the LASS function for the scalar contribution, which is given  
 154 by (see Ref. [10])

$$F_S = \lambda A_s, A_s = \frac{\sqrt{s}}{P} (\sin \delta_B e^{i\delta_B} + e^{2i\delta_B} BW_{K^*(1430)}), \quad (13)$$

155 where  $\lambda$  is a real constant, and  $P$  is defined in Eq. 11. The phase  $\delta_B$  is defined by the equation:

$$\cot \delta_B = \frac{1}{aP} + \frac{bP}{2} \quad (14)$$

156 As will be shown below, the description of the data when using the LASS function is significantly  
 157 worse than when using Eq. 12.

## 158 5.2 Response matrix

159 To predict the number of entries in each bin of the observed mass distribution one requires the  
 160 response matrix  $R_{ij}$ , which reflects the limited efficiency and resolution of the detector. This is  
 161 obtained from the Monte Carlo, which we use to produce a two-dimensional distribution of the  
 162 measured minus true hadronic mass,

$$x = m_{\text{meas}} - m_{\text{true}}, \quad (15)$$

163 versus the true mass. We then split the distribution into vertical slices and project onto the vertical  
 164 (measured minus true mass) axis. These distributions represent the detector response corresponding  
 165 to the true mass of each slice. The distributions from each slice were fitted with a parametric pdf,  
 166 and the resulting parameters were themselves parametrized as a function of the true mass. The  
 167 slices were chosen such that all of the projected distributions have approximately the same number  
 168 of events to avoid spurious fit results due to extremely small numbers of events.

169 The tails of the distributions of  $x = m_{\text{meas}} - m_{\text{true}}$  were seen to be longer than those which a  
 170 Gaussian pdf could accurately fit, but a good description was obtained with a Student's  $t$  distribu-  
 171 tion,

$$f(x; \mu, \lambda, \nu) = \frac{1}{\lambda} \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi} \Gamma(\nu/2)} \left(1 + \frac{t^2}{\nu}\right)^{-(\nu+1)/2}, \quad (16)$$

172 where

$$t = \frac{x - \mu}{\lambda}. \quad (17)$$

173 The Student's  $t$  distribution is flexible enough to account for the long tails because of its parameter  
 174  $\nu$ , which in effect controls the extent of the tails. As  $\nu$  goes to infinity the tail approaches Gaussian  
 175 form, and if  $\nu$  is 1, then the tail is that of a Cauchy distribution.

176 Figure 4 shows the fitted parameters using the Student's  $t$  distribution in each of the 40 slices  
 177 versus the true mass. As can be seen in Figs. 4(a) and (b), the values of  $\nu$  and  $\lambda$  exhibit a non-  
 178 negligible dependence on the true mass, and these are fitted with a linear function. A linear fit was  
 179 also carried out for the central value of the response function,  $\mu$ , which showed that the values are  
 180 to high accuracy consistent with zero. Therefore in the following we constrain  $\mu$  to be zero.

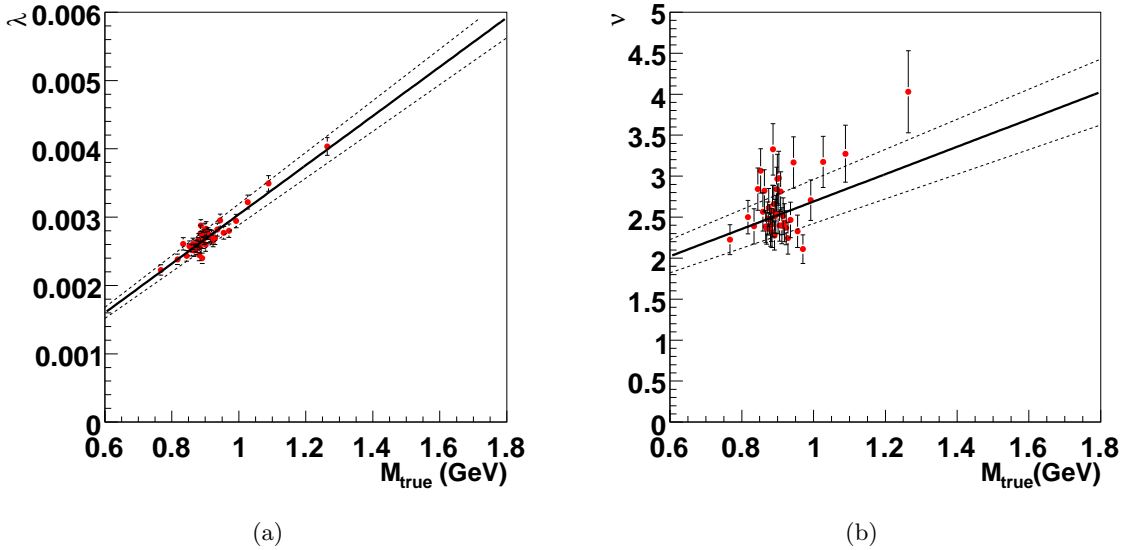


Figure 4: Fitted parameter values of (a) the scale parameter  $\lambda$  and (b) the number of degrees of freedom  $\nu$  of the Student's  $t$  distribution, versus true mass. The dashed lines indicated the variations in parameter values which were employed to assess systematic uncertainties.

181 The fitted values of  $\nu$  and  $\lambda$  as a function of the true mass are then used to obtain the response  
 182 matrix element  $R_{ij}$  for all values of the measured and true mass (i.e., all  $i$  and  $j$ ) using

$$\begin{aligned}
 R_{ij} &= P(\text{found in bin } i | \text{true value in bin } j) \\
 &= P(\text{found in bin } i | \text{found somewhere})P(\text{found somewhere} | \text{true value in bin } j) \\
 &= \int_{\text{bin } i} f(x; \vec{\theta}(m_{\text{true},j})) dx \varepsilon_j,
 \end{aligned}
 \tag{18}$$

183 where  $\varepsilon_j$  is the efficiency for bin  $j$  and  $\vec{\theta} = (\lambda, \nu)$  represents the set of parameters that were fitted  
 184 using the distributions of  $x$  from the individual slices.

### 5.3 Maximum-Likelihood fit

For our primary analysis we use a binned extended Maximum-Likelihood fit, where the number of events  $n_i$  in bin  $i$  is modeled as a Poisson distributed quantity.

The analysis requires the expected background given by the parameters  $\beta_i$  in (3). These are obtained from Monte Carlo, and thus their estimated values have a statistical error. In a least-squares fit, these errors can be taken into account by modifying the denominator of the expression to be minimised (see Section 5.4). In a binned maximum-likelihood fit, this is not possible. A method to include the statistical uncertainty arising from using finite MC samples was proposed by Barlow and Beeston [12], which was adapted to our problem. Strictly speaking one should use a binomial model for  $m_{ij}$ , but we approximate  $m_{ij}$  as a Poisson variable because the number of background events in each bin for each component is small compared to the total number of generated MC events.

For a given background component  $j$ , for each bin  $i$ , let the expected number of events be  $\beta_{ij}$ . The Monte Carlo sample for this background mode gives a number of events  $m_{ij}$  observed in the corresponding bin. The expectation value of  $m_{ij}$  is related to  $\beta_{ij}$  by

$$E[m_{ij}] = \tau_j r_j \beta_{ij}, \quad (19)$$

where  $\tau_j$  is a scale factor that relates the luminosity of the MC sample for mode  $j$  to that of the data, and  $r_j$  is a factor that allows for the uncertainty in the prediction of the rate of the background process. The best estimate of  $r_j$  is equal to unity, but this is treated as a Gaussian distributed quantity with a standard deviation equal to the relative uncertainty on the production rate for the  $j$ th background mode.

The uncertainties in the values of other nominally fixed model parameters, e.g., the resonance parameters of the  $K^*(1410)$ , can be incorporated into the fit in a similar way. For a given parameter  $\eta$  one has a previously estimated value  $\hat{\eta}$  and standard deviation  $\sigma_\eta$ , taken, e.g., from the PDG. One includes in the likelihood function a Gaussian term in  $\eta$  centered about  $\hat{\eta}$  with a standard deviation  $\sigma_\eta$ , and regards  $\eta$  as an adjustable parameter. In the nominal fit presented below, this procedure is applied only for the mass and width of the  $K^*(1410)$  (referred to below as  $\eta_1$  and  $\eta_2$ ).

We also include in the likelihood function terms which account for the uncertainty in the shapes of background mass distributions. From Fig. 7(a) one can see that the contributions from tau-lepton decays to  $K_S^0\pi^-K_L^0$  and  $K_S^0\pi^-\pi^0$  both have a peaking structure at the mass of the  $K^*(892)$ , and thus any uncertainty in the modeling of these modes will lead to a systematic uncertainty in the measurement of the  $K^*(892)$  mass and width. This is particularly true for the  $K_S^0\pi^-K_L^0$  mode, as it makes a larger contribution and the information on its shape is based largely on lower-statistics measurements from LEP. The mass and width of the  $K^*(892)$  for the MC simulation of this mode were taken to be the nominal PDG values, which differ by about 3 and 5 MeV, respectively, from the values found in the present analysis. Therefore we assign a 3 MeV uncertainty to the mean and 5 MeV uncertainty to the width of the  $K_S^0\pi^-K_L^0$  background contribution.

To propagate these uncertainties into the full fit of the  $K_S^0\pi^-$  mass distribution, we introduce two additional adjustable parameters,  $\vec{\alpha} = (\alpha_1, \alpha_2)$ , which have the effect of shifting and stretching the shape of the distribution. This transformation is applied to the  $\beta_{ij}$  values for the  $K_S^0\pi^-K_L^0$  background mode and the altered values are then used in the likelihood function. The mean and standard deviation of the initial MC template are prevented from changing too much by regarding their nominal values as Gaussian distributed estimates with the assigned uncertainties of  $\sigma_{\mu_{KK\pi}} = 3 \text{ MeV}$  and  $\sigma_{\sigma_{KK\pi}} = 5 \text{ MeV}$ , respectively. More details on the mathematical procedure

228 are given in Ref. [13].

229 Applying this procedure to the  $K_S^0\pi^-K_L^0$  background mode led to very small increases in the  
 230 errors of the mass and width of the  $K^*(892)$ . As the next largest background contribution from  
 231  $K_S^0\pi^-\pi^0$  is substantially smaller and furthermore its shape was directly measured specifically for  
 232 use in the present analysis, the effect of its systematic uncertainty is not considered to make a  
 233 significant contribution. Therefore we only include the extra parameters  $\alpha_1$  and  $\alpha_2$  for the  $K_S^0\pi^-K_L^0$   
 234 background.

235 Putting together these ingredients we can write the full likelihood function as the following  
 236 product of Poisson terms for  $n_i$  and  $m_{ij}$  and Gaussian terms for  $r_j$ , the mass and width of the  
 237  $K^*(1410)$ , and for the mean and standard deviation of the  $K_S^0\pi^-K_L^0$  background:

$$\begin{aligned}
 L(\mu_{\text{tot}}, \vec{\theta}, \vec{\beta}, \vec{r}, \vec{\eta}, \vec{\alpha}) &= \prod_{i=1}^N \frac{\nu_i^{n_i}}{n_i!} e^{-\nu_i} \times \prod_{i=1}^N \prod_{j=1}^{N_{\text{bkg}}} \frac{(\tau_j r_{ij} \beta_{ij})^{m_{ij}}}{m_{ij}!} e^{-\tau_j r_{ij} \beta_{ij}} \\
 &\times \prod_{j=1}^{N_{\text{bkg}}} \frac{1}{\sqrt{2\pi} \sigma_{r_j}} e^{-(r_j-1)^2/2\sigma_{r_j}^2} \\
 &\times \prod_k \frac{1}{\sqrt{2\pi} \sigma_{\eta_k}} \exp\left(-\frac{1}{2} \frac{(\eta_k - \hat{\eta}_k)^2}{\sigma_{\eta_k}^2}\right) \\
 &\times \frac{1}{\sqrt{2\pi} \sigma_{\mu_{KK\pi}}} \exp\left(-\frac{1}{2} \frac{(\mu_{KK\pi}(\vec{\alpha}) - \mu_{KK\pi}(0))^2}{\sigma_{\mu_{KK\pi}}^2}\right) \\
 &\times \frac{1}{\sqrt{2\pi} \sigma_{\sigma_{KK\pi}}} \exp\left(-\frac{1}{2} \frac{(\sigma_{KK\pi}(\vec{\alpha}) - \sigma_{KK\pi}(0))^2}{\sigma_{\sigma_{KK\pi}}^2}\right). \quad (20)
 \end{aligned}$$

238 Here  $\mu_{KK\pi}(0)$  and  $\sigma_{KK\pi}(0)$  are the nominal mean and standard deviation of the  $K_S^0\pi^-K_L^0$  back-  
 239 ground, and  $\mu_{KK\pi}(\vec{\alpha})$  and  $\sigma_{KK\pi}(\vec{\alpha})$  are the values corresponding to the parameters  $\vec{\alpha} = (\alpha_1, \alpha_2)$ .  
 240 The parameters  $\mu_{\text{tot}}$ ,  $\vec{\theta}$ ,  $\vec{\beta}$ ,  $\vec{r}$  and  $\vec{\alpha}$  enter through equations (3), (6) and (19).

241 Instead of maximizing  $L$  we minimize the equivalent quantity proportional to  $-2 \ln L$ , normal-  
 242 ized in such a way that the minimized value behaves like a chi-square distributed goodness-of-fit  
 243 statistic (see, e.g., [9]):

$$\begin{aligned}
 \chi^2(\mu_{\text{tot}}, \vec{\theta}, \vec{\beta}, \vec{r}, \vec{\eta}, \vec{\alpha}) &= 2 \sum_{i=1}^N n_i \ln \frac{n_i}{\nu_i(\vec{\theta})} + \nu_i(\vec{\theta}) - n_i \\
 &+ 2 \sum_{i=1}^N \sum_{j=1}^{N_{\text{bkg}}} m_{ij} \ln \frac{m_{ij}}{\tau_j r_{ij} \beta_{ij}} + \tau_j r_{ij} \beta_{ij} - m_{ij} \\
 &+ \sum_{j=1}^{N_{\text{bkg}}} \left( \frac{r_j - 1}{\sigma_{r_j}} \right)^2 + \sum_k \left( \frac{\eta_k - \hat{\eta}_k}{\sigma_{\eta_k}} \right)^2 \\
 &+ \left( \frac{\mu_{KK\pi}(\vec{\alpha}) - \mu_{KK\pi}(0)}{\sigma_{\mu_{KK\pi}}} \right)^2 + \left( \frac{\sigma_{KK\pi}(\vec{\alpha}) - \sigma_{KK\pi}(0)}{\sigma_{\sigma_{KK\pi}}} \right)^2. \quad (21)
 \end{aligned}$$

244 In Eq. (21) the logarithmic terms are taken as zero if the observed number of events ( $n_i$  or  $m_{ij}$ ) is  
 245 zero. It can be shown that for all  $n_i$  and  $m_{ij}$  sufficiently large, the sampling distribution of of this  
 246 quantity approaches a chi-square distribution for  $N_{\text{meas}} - n_{\text{par}}$  degrees of freedom, where  $N_{\text{meas}}$  is  
 247 the number of measured values and  $n_{\text{par}}$  is the number of adjustable parameters. Only a few of  
 248 the bins in the measured distribution have less than 10 entries, and therefore the minimized value  
 249 of (21) provides a measure of goodness-of-fit that is to good approximation equivalent to the usual  
 250 chi-square statistic.

251 To determine the number of degrees of freedom, not only the  $n_i$  but also the  $m_{ij}$  count effectively  
 252 as measured values, as do the estimated of values of unity for each of the  $r_j$ , the nominal  $K^*(1410)$   
 253 mass, and the nominal values of the mean and standard deviation of the  $K_S^0\pi^-K_L^0$  background.  
 254 Thus the inclusion of the parameters  $\beta_{ij}$ ,  $r_j$ ,  $\eta_1, \eta_2$ ,  $\alpha_1$  and  $\alpha_2$  are compensated by corresponding  
 255 values that are treated as measurements and there is no resulting change in the number of degrees  
 256 of freedom; for the nominal fit this is 88.

## 257 5.4 Least-Squares fit

258 It is useful to also perform a Least-Squares(LS) fit to the data which can serve as an important  
 259 cross-check of our final parameter values. Each of the background components is subtracted from  
 260 the data, yielding measured values

$$y_i = n_i - \sum_j \frac{m_{ij}}{r_j \tau_j} \quad (22)$$

261 where  $r_j$  and  $\tau_j$  are defined in Section 5.3 above. The expected number of events is found (cf.  
 262 equation (3)) using

$$\lambda_i = \sum_j R_{ij} \mu_j . \quad (23)$$

263 We then minimize

$$\begin{aligned} \chi^2(\mu_{\text{tot}}, \vec{\theta}, \vec{r}, \vec{\eta}, \vec{\alpha}) &= \sum_{i=1}^N \frac{(y_i - \lambda_i)^2}{\sigma_i^2} + \sum_{j=1}^{N_{\text{bkg}}} \left( \frac{r_j - 1}{\sigma_{r_j}} \right)^2 + \sum_k \left( \frac{\eta_k - \hat{\eta}_k}{\sigma_{\eta_k}} \right)^2 \\ &+ \left( \frac{\mu_{KK\pi}(\vec{\alpha}) - \mu_{KK\pi}(0)}{\sigma_{\mu_{KK\pi}}} \right)^2 + \left( \frac{\sigma_{KK\pi}(\vec{\alpha}) - \sigma_{KK\pi}(0)}{\sigma_{\sigma_{KK\pi}}} \right)^2 , \end{aligned} \quad (24)$$

264 where  $\sigma_i^2$  is the estimated value of the variance of  $y_i$ . This is given by

$$V[y_i] = V \left[ n_i - \sum_j \frac{m_{ij}}{r_j \tau_j} \right] = V[n_i] + \sum_j \left( \frac{1}{r_j \tau_j} \right)^2 V[m_{ij}] \quad (25)$$

265 For the denominator used in the LS fit this can be estimated by

$$\sigma_i^2 = n_i + \sum_j \left( \frac{1}{r_j \tau_j} \right)^2 m_{ij} . \quad (26)$$

266 **5.5 FIT RESULTS FOR  $K_S^0\pi^-$  MASS DISTRIBUTION**

267 The fitting procedures described above have been carried out using a variety of hypotheses. Figures  
268 5 to 7 show our Least-Squares fits, while Figure 8 is performed using the method of Maximum-  
269 Likelihood.

270 (a) single  $K^*(892)$

271 (b)  $K^*(892) + K^*(1410)$

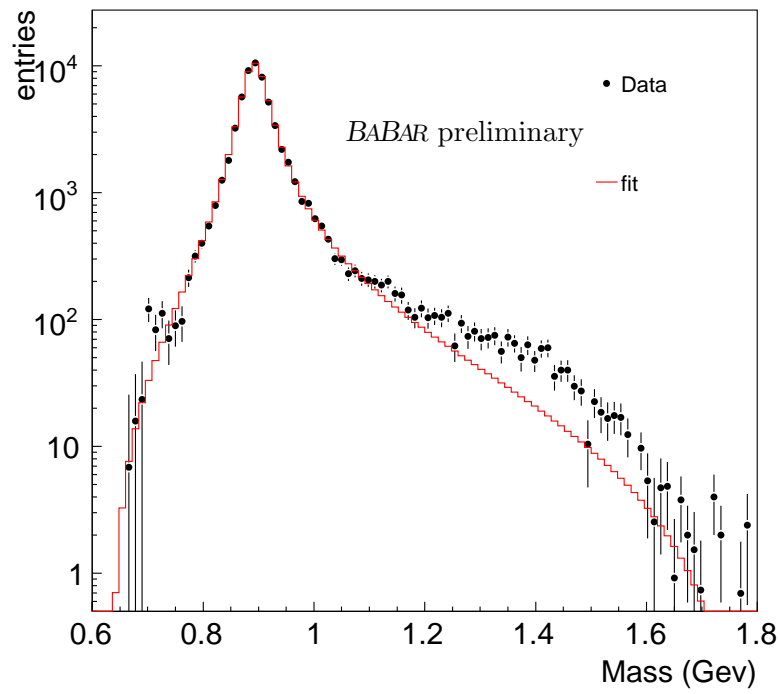
272 (c)  $K^*(800) + K^*(892) + K^*(1410)$

273 (d)  $K^*(800) + K^*(892) + K^*(1430)$

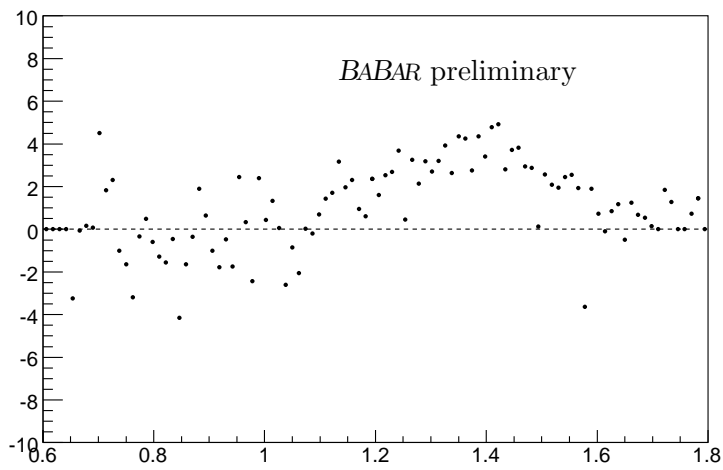
274 (e)  $K^*(800) + K^*(892) + K^*(1680)$

275 (f)  $K^*(892) + \text{LASS}$

276 Results for the fits are shown in Figs. 5 through 8 and in Table 4. As described earlier the fitted  $r$   
277 parameters,(and  $\alpha_1, \alpha_2$  for the  $K_S^0 K_L^0 \pi^-$ ) determine a new shape for the background, and it is this  
278 new background shape which is subtracted from the data. As such each background subtracted  
279 data histogram will look different for each fit.

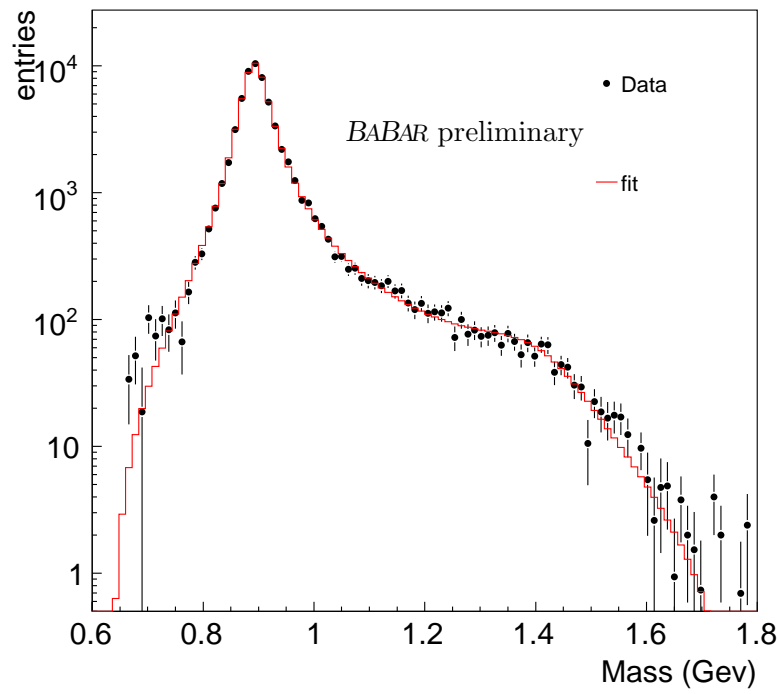


(a)

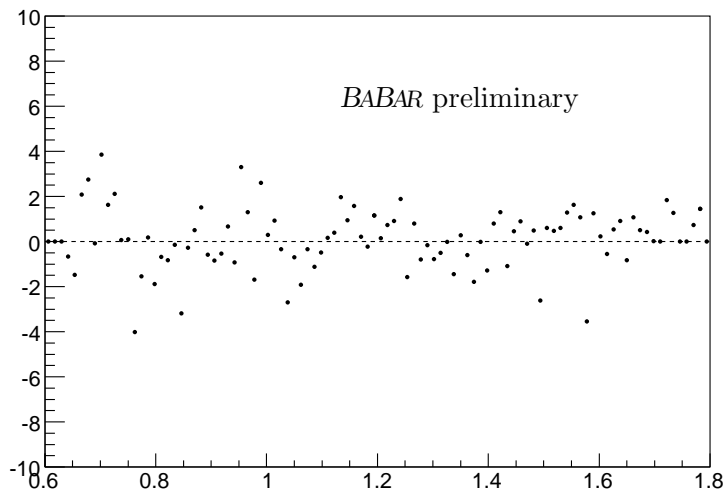


(b)

Figure 5: (a)  $K_S^0\pi^-$  invariant mass distribution fit using only the  $K^*(892)$  resonance, and (b) the measured minus fitted values divided by the measurement errors



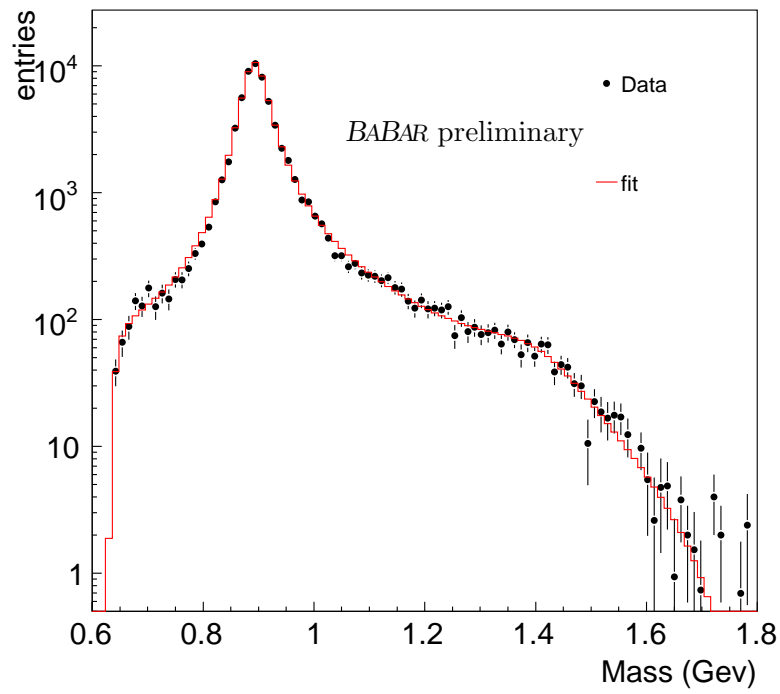
(a)



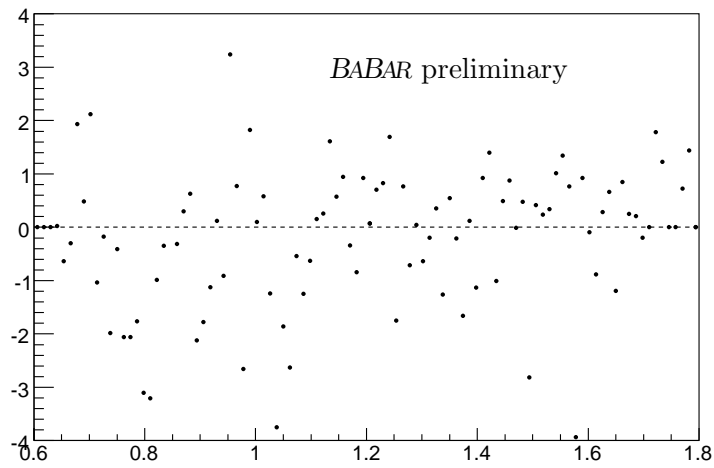
(b)

Figure 6: (a)  $K_S^0\pi^-$  invariant mass distribution fit using a  $K^*(892) + K^*(1410)$ , and (b) the measured minus fitted values divided by the measurement errors



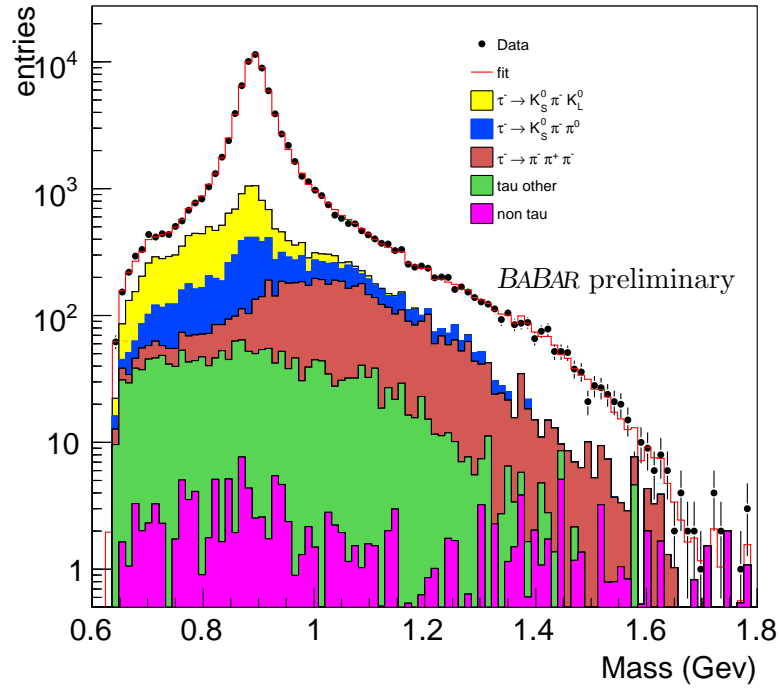


(a)

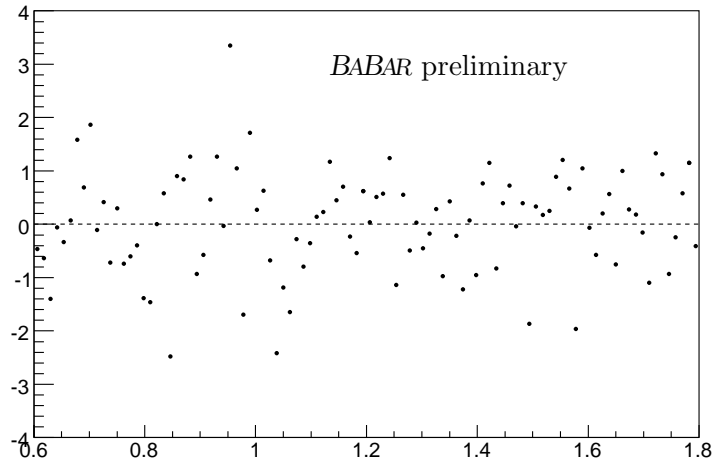


(b)

Figure 7: (a)  $K_S^0\pi^-$ -invariant mass distribution fit using  $K^*(800) + K^*(892) + K^*(1410)$ , and (b) the measured minus fitted values divided by the measurement errors



(a)



(b)

Figure 8: (a) Fit of  $K_S^0\pi^-$  invariant mass distribution without background subtracted, using  $K^*(892)$  and  $K^*(1410)$  and  $K^*(800)$ , and (b) the measured minus fitted values divided by the measurement errors. The method of maximum likelihood is used to perform this fit.

### 280 5.5.1 Discussion of fits

281 Figure 5(a) shows that a single  $K^*(892)$  is clearly not enough to model the mass spectrum accurately.  
 282 This was seen by the Belle collaboration [3], which proposed that the distribution should contain  
 283 contributions from a  $K^*(800)$  scalar and  $K^*(1410)$  vector resonances.

284 In the region around 1.4 GeV in Fig. 5(a), the data are significantly higher than the fitted  
 285 curve. As can be seen in Fig. 6, the addition of the  $K^*(1410)$  gives a significant improvement  
 286 to the high mass region, yielding a  $\chi^2$  of 130.04 for 95 degrees of freedom. In these fits the rate  
 287 of the  $K^*(1410)$  was allowed to vary within the error given in the PDG. However, to enable this  
 288 two-resonance fit model to accurately fit the low mass region, the background in this area has to  
 289 be distorted. This can be seen by the fitted  $r$  values from our model, shown in Table 3. Columns  
 290 (a) and (b) show the values for our one resonance and two resonance models. The default value of  
 291  $r$  is 1 for all background modes; where  $r$  is less than 1 indicates that the fit has had to increase the  
 292 rate of this mode by a factor  $1 - r$ . This is especially prominent in the  $\tau \rightarrow K_S^0 K_L^0 \pi^- \nu_\tau$  mode.

293 The inclusion of the  $K^*(800)$  further reduces our  $\chi^2$  to 113.05 for 94 degrees of freedom. This  
 294 is a significantly better goodness-of-fit value than our  $K^*(892) + K^*(1410)$  fit model. For the mass  
 295 and width of the  $K^*(800)$  we use the measurements from the BES collaboration [14]. The result  
 296 is shown in Fig. 7(a). In addition the  $r$  values for this fit, shown in column (c) in Table 3, are  
 297 closer to 1, indicating the fit is not having to vary these backgrounds by much to fit the total mass  
 298 spectrum. This can also be seen by considering the background in the low mass region in Figs. 5(a)  
 299 to 7(a), where there is significantly less background in the one and two resonance fits compared  
 300 to the fit including the  $K^*(800)$ .

Table 3: Table of fitted  $r$  parameters (see text). Column (a) refers to a fit using only a  $K^*(892)$ ,  
 column (b) refers to a fit using  $K^*(892) + K^*(1410)$  and column (c) refers to a fit using  $K^*(800)$   
 +  $K^*(892) + K^*(1410)$ .

	(a)	(b)	(c)
$\tau \rightarrow K_S^0 K_L^0 \pi^- \nu_\tau$	$0.852 \pm 0.034$	$0.773 \pm 0.030$	$1.134 \pm 0.078$
$\tau \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$	$0.977 \pm 0.050$	$1.00754 \pm 0.047$	$1.037 \pm 0.046$
$\tau \rightarrow \pi^+ \pi^- \pi^- \nu_\tau$	$0.982 \pm 0.0086$	$0.9998 \pm 0.0088$	$1.0008 \pm 0.0088$
$\tau \rightarrow \text{other}$	$0.953 \pm 0.033$	$0.986 \pm 0.033$	$0.995 \pm 0.032$
non $\tau$ background	1.00000	1.00000	1.00000

301 The  $\tau \rightarrow K_S^0 K_L^0 \pi^- \nu_\tau$  mode is based on Monte Carlo form factors derived from a low statistics  
 302 ALEPH measurement [15]. It is therefore plausible that this is not an accurate representation  
 303 of the shape of this mode. In order to ascertain by how much the  $\tau \rightarrow K_S^0 K_L^0 \pi^- \nu_\tau$  background  
 304 would have to vary to completely account for the low mass shoulder, in the absence of a  $K^*(800)$ ,  
 305 we repeated the fit with a  $K^*(892) + K^*(1410)$  and removed the penalty term for the mean and  
 306 standard deviation of the  $\tau \rightarrow K_S^0 K_L^0 \pi^- \nu_\tau$  background in the  $\chi^2$  minimisation. This would enable  
 307 the fit to have a larger parameter space in which to find the minimum. The result of this fit yields  
 308 a  $\chi^2/\text{dof} = 115.7/92$  which is very close to our nominal fit value, which indicates that this model  
 309 can in principle provide a good description of the data. However the  $\tau \rightarrow K_S^0 K_L^0 \pi^- \nu_\tau$  has to be  
 310 distorted by a significant amount, which can be seen in Figure 9(a). It should be noted that when  
 311 one considers the mass spectrum obtained by ALEPH, it exhibits large experimental uncertainties,  
 312 and therefore our distorted histogram is reasonably consistent with the ALEPH result [15]. It is  
 313 therefore not possible at this stage to comment firmly on the existence or necessity of a  $K^*(800)$

314 in our mass spectrum. Further studies on the mass spectrum of the  $\tau \rightarrow K_S^0 K_L^0 \pi^- \nu_\tau$  background  
 315 mode are ongoing to confirm that the Monte Carlo is accurate.

316 If instead of a  $K^*(1410)$  one uses a scalar  $K^*(1430)$ , one finds a comparable  $\chi^2$  value of 114.11  
 317 for 94 degrees of freedom. As such the  $K^*(1410)$  and  $K^*(1430)$  cannot be differentiated on their  
 318  $\chi^2$  value.

319 Using the LASS function for the scalar form factor gives a  $\chi^2$  value of 157.7 for 94 degrees of  
 320 freedom, i.e., significantly worse than our nominal fit.

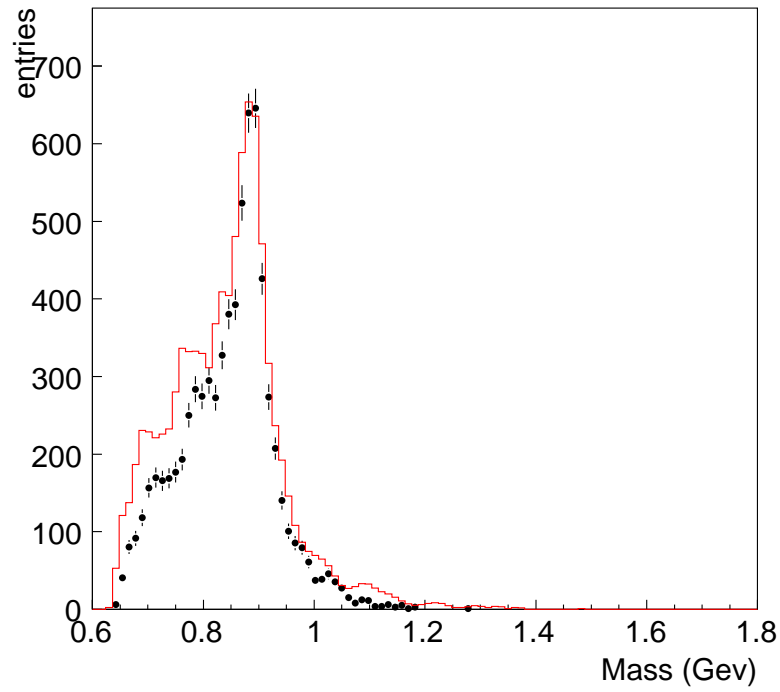
321 We also present a fit using the  $K^*(1680)$  instead of the  $K^*(1410)$ , as a check of the similar  
 322 exercise carried out by Belle. Using only this resonance for the high-mass region, however, does  
 323 not provide an adequate description of the data.

324 To summarize, the fit using only the two resonances  $K^*(892)$  and  $K^*(1410)$  is able to describe  
 325 the low-mass region of the distribution, if the modeling of the background were to be wrong by  
 326 a significant amount. This is currently being investigated. The model using  $K^*(800) + K^*(892)$   
 327  $+ K^*(1410)$  gives the best goodness-of-fit based on the  $\chi^2$  value, and so this is chosen to be our  
 328 preliminary nominal fit. The difference between the parameter values of these two fit models are  
 329 used as systematic uncertainties. This is discussed further in the next section. The resulting values  
 330 for the mass and width of the  $K^*(892)$  are found to be

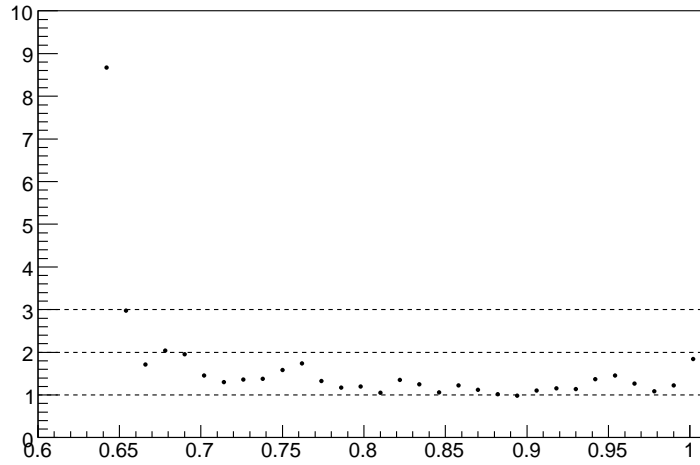
$$M(K^*(892)^-) = 894.57 \pm 0.19 \text{ (stat.) MeV} , \quad (27)$$

$$\Gamma(K^*(892)^-) = 45.89 \pm 0.43 \text{ (stat.) . MeV} \quad (28)$$

331 The statistical errors quoted already cover a number of systematic uncertainties such as those  
 332 in the rates and shapes of backgrounds, which were incorporated by including corresponding ad-  
 333 justable parameters in the fit. Several additional sources of systematic uncertainty are discussed  
 334 in Section 5.6.



(a)



(b)

Figure 9: (a) The original Monte Carlo histogram for  $\tau \rightarrow K_S^0 K_L^0 \pi^- \nu_\tau$  (points), and the distortion needed to account for the low mass shoulder (red line). (b) The ratio of the distorted histogram to the Monte Carlo histogram for the low mass shoulder.

Table 4: Table of Fitted parameters: The inclusion of different resonances are denoted by the symbol  $\oplus$ . Square brackets [ ] around the paramters denote that they have been fixed in the fit. Curly brackets { } around the parameters denote that they have been constrained in the fit. Column c is the nominal fit

Scenario	a	b	c	d	e	f
Resonances	$K^*(892) \oplus$	$K^*(892) \oplus$ [ $K^*(1410)$ ]	[ $K^*(800)$ ] $\oplus$ $K^*(892) \oplus$ { $K^*(1410)$ }	[ $K^*(800)$ ] $\oplus$ $K^*(892) \oplus$ [ $K^*(1430)$ ]	[ $K^*(800)$ ] $\oplus$ $K^*(892) \oplus$ [ $K^*(1680)$ ]	$K^*(892) \oplus$  LASS
$M(892)$ (MeV/ $c^2$ )	894.544 $\pm$ 0.171	894.412 $\pm$ 0.187	894.565 $\pm$ 0.193	894.673 $\pm$ 0.193	894.393 $\pm$ 0.184	894.855 $\pm$ 0.196
$\Gamma(892)$ (MeV)	47.673 $\pm$ 0.437	46.206 $\pm$ 0.455	45.893 $\pm$ 0.434	45.834 $\pm$ 0.426	45.491 $\pm$ 0.392	47.022 $\pm$ 0.452
$ \beta $	N/A	0.095 $\pm$ 0.006	0.075 $\pm$ 0.007	N/A	N/A	
$\arg(\beta)$	N/A	1.983 $\pm$ 0.139	1.747 $\pm$ 0.18	N/A	N/A	
$M(1410)$ (MeV/ $c^2$ )	N/A	{1434.23 $\pm$ 11.19 (PDG)}	{1425.55 $\pm$ 12.47 (PDG)}	N/A	N/A	N/A
$\Gamma(1410)$ (MeV)	N/A	{253.80 $\pm$ 17.68 (PDG)}	{238.76 $\pm$ 18.85 (PDG)}	N/A	N/A	N/A
$ \lambda (1430)$	N/A	N/A	N/A	5.059 $\pm$ 0.311		N/A
$\arg(\lambda)(1430)$	N/A	N/A	N/A	8.670 $\pm$ 0.244		N/A
$M(1430)$ (MeV/ $c^2$ )				[1425 $\pm$ 50(PDG)]		
$\Gamma(1430)$ (MeV)				[270 $\pm$ 80(PDG)]		
$ \gamma (1680)$					0.199 $\pm$ 0.016	
$\arg(\gamma)(1680)$					3.559 $\pm$ 0.184	
$M(1680)$ (MeV/ $c^2$ )	N/A	N/A	N/A	N/A	[1717 $\pm$ 27 (PDG)]	
$\Gamma(1680)$ (MeV)	N/A	N/A	N/A	N/A	[322 $\pm$ 110 (PDG)]	
$M(800)$ (MeV/ $c^2$ )	N/A	N/A	[841 $\pm$ 30 $^{+81}_{-73}$ (BES)]	[841 $\pm$ 30 $^{+81}_{-73}$ (BES)]	[841 $\pm$ 30 $^{+81}_{-73}$ (BES)]	
$\Gamma(800)$ (MeV)	N/A	N/A	[618 $\pm$ 90 $^{+96}_{-144}$ (BES)]	[618 $\pm$ 90 $^{+96}_{-144}$ (BES)]	[618 $\pm$ 90 $^{+96}_{-144}$ (BES)]	
$\varkappa$	N/A	N/A	1.938 $\pm$ 0.11	0.255 $\pm$ 0.019	2.237 $\pm$ 0.101	
$\chi^2$	399.778	130.044	113.049	119.108	144.711	148.375
# d.o.f.	97	95	94	94	94	94
$\chi^2/\#$ d.o.f.	4.121	1.369	1.203	1.267	1.539	1.579
Prob.( $\chi^2$ )	<0.0001	0.0098	0.0880	0.0411	0.0006	0.0002

## 5.6 SYSTEMATIC UNCERTAINTIES

Several important sources of systematic uncertainty are already covered in the fit by including in the model corresponding adjustable parameters, as described in Section 5.3. These include the uncertainty due to limited Monte Carlo statistics, total background rates, and the shape of the  $K_S^0\pi^-K_L^0$  background.

Two additional sources of systematic uncertainty in the  $\tau^- \rightarrow K_S^0\pi^-\nu_\tau$  analysis are from the response matrix and the choice of fit model. These are discussed in Sections 5.6.1 – 5.6.2 below and summarized in Section 5.6.3.

### 5.6.1 Uncertainty in the response matrix

The response matrix  $R_{ij}$  is derived from the Monte Carlo simulation of the detector. As described in Section 5.2, the response matrix was parameterized using a Student's  $t$  distribution with an adjustable scale parameter  $\lambda$  and number of degrees of freedom  $\nu$ , each of which were themselves fitted as a linear function of the true hadronic mass.

As a conservative estimate of the uncertainty of the detector response, which is dominated by modeling of the tracking and Calorimeter, we have varied  $\lambda$  by  $\pm 5\%$  and  $\nu$  by  $\pm 10\%$  relative to its nominal fitted value. Varying  $\lambda$  results in a 0.18 MeV change in the width of the  $K^*(892)$  and a 0.023 MeV change in the mass. Varying  $\nu$  results in a 0.28 MeV change in the width of the  $K^*(892)$  and a 0.030 MeV change in the mass.

### 5.6.2 Uncertainty due to choice of fit model

As a check of the fitting method we have taken a fully reconstructed Monte Carlo sample of signal events, and fitted them using the signal model. As the MC generator models the  $\tau^- \rightarrow K_S^0\pi^-\nu_\tau$  decay with only the  $K^*(892)$  resonance, the fit model also only contained this resonance. The fit gives values for the mass and width of the  $K^*(892)$  of

$$\begin{aligned} M(K^*(892)^-) &= 891.924 \pm 0.143 \text{ (stat.)}, \\ \Gamma(K^*(892)^-) &= 51.138 \pm 0.326 \text{ (stat.)}. \end{aligned}$$

The  $K^*(892)$  mass and width values in the generator are

$$\begin{aligned} M(K^*(892)^-) &= 891.660, \\ \Gamma(K^*(892)^-) &= 50.800. \end{aligned}$$

The difference between the input and fitted values are 0.264 MeV and 0.338 MeV for the mass and width respectively. We take this difference and apply a correction as an additive shift in our nominal values. This changes our nominal fit values to 894.301 MeV and 45.555 MeV for the mass and width respectively of the  $K^*(892)$ . The statistical errors on each parameter are then taken as systematic uncertainties and are added in quadrature to the other systematic uncertainties.

A further uncertainty in the fit model stems from the choice of resonances to include in the fit. Although using the  $K^*(1430)$  instead of the  $K^*(1410)$  resulted in a small increase in the  $\chi^2$ , the model provides nevertheless a good qualitative fit to the data. The fit model which uses just a  $K^*(892)$  and  $K^*(1410)$  is also able to provide a good fit, albeit with a significantly distorted

368 background. Therefore we take the difference in the mass and width values for the  $K^*(892)$  between  
 369 our nominal fit and the alternative models which also yield comparable  $\chi^2$  values, as a source of  
 370 systematic uncertainty. The largest discrepancy in parameter values came from our two resonance  
 371 model, which yielded  $K^*(892)$  mass and width values of 894.447 MeV and 46.223 MeV respectively.  
 372 The results of our systematic studies are summarized in Table 5.

### 373 5.6.3 Summary of systematic uncertainties

374 Table 5 summarizes the systematic uncertainties for the fit of the measurements of the mass and  
 375 width of the  $K^*(892)$ .

Table 5: Table of systematic uncertainties in the mass and width of the  $K^*(892)$  (see text).

	M( $K^*(892)$ )	$\Gamma(K^*(892))$
Response matrix width ( $\pm 5\%$ variation of $\lambda$ )	0.023	0.180
Response matrix tails ( $\pm 10\%$ variation of $\nu$ )	0.030	0.280
Statistical error on fit to reconstructed MC	0.143	0.326
Without using $K^*(800)$	0.118	0.330
Total systematic (quadratic sum)	0.189	0.571

## 376 6 SUMMARY AND CONCLUSIONS

377 We have carried out studies of the decays  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  and  $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$  using  $384.6 \text{ fb}^{-1}$  of  
 378  $e^+e^-$  collision data provided by the PEP-II accelerator, operating primarily at  $\sqrt{s} = 10.58 \text{ GeV}$ ,  
 379 and recorded using the *BABAR* detector. We have measured the branching ratios for  $\tau^- \rightarrow$   
 380  $K_S^0 \pi^- \pi^0 \nu_\tau$ , which is found to be

$$\mathcal{B}(\tau^- \rightarrow K^0 \pi^- \pi^0 \nu_\tau) = (0.342 \pm 0.006 (\text{stat.}) \pm 0.015 (\text{sys.}))\% .$$

381 The measurement of  $\mathcal{B}(\tau^- \rightarrow K^0 \pi^- \nu_\tau)$  is fully consistent with the preliminary result presented  
 382 by the BaBar collaboration in 2008 [1]. It is also in good agreement with the value found by the  
 383 Belle collaboration [3] and has a slightly smaller total uncertainty. For  $\mathcal{B}(\tau^- \rightarrow K^0 \pi^- \pi^0 \nu_\tau)$  the  
 384 value obtained is also consistent with the present world average and represents an improvement in  
 385 accuracy by a factor of 2.

386 For the  $\tau^- \rightarrow K_S^0 \pi^- \pi^0 \nu_\tau$  mode we have measured the mass distributions of different combina-  
 387 tions of final state hadrons:  $\pi^- \pi^0$ ,  $K_S^0 \pi^-$ ,  $K_S^0 \pi^0$  and  $K_S^0 \pi^- \pi^0$ . These were used to make important  
 388 improvements to the TAUOLA Monte Carlo generator, which allowed for a precise estimation of the  
 389 background contribution from this mode in the analysis of the  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$  channel.

390 We have carried out a fit of the hadronic mass distribution for  $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$ . Including a  
 391 systematic shift in our parameter values originating from an estimated reconstruction bias, (see  
 392 section 5.6.2), this yields precise measurements for the mass and width of the  $K^*(892)$  resonance:

$$M(K^*(892)^-) = 894.30 \pm 0.19 (\text{stat.}) \pm 0.19 (\text{syst.}) \text{ MeV} ,$$

$$\Gamma(K^*(892)^-) = 45.56 \pm 0.43 (\text{stat.}) \pm .57 (\text{syst.}) \text{ MeV} .$$



393 These values confirm the Belle collaboration's measurements [3] that indicated a  $K^*(892)$  mass  
 394 several MeV higher and a width several MeV lower than the world average. The results reported  
 395 here represent a factor of two improvement in accuracy relative to the Belle measurements.

396 We analyse the possibility of other resonances being present in this mass spectrum, and conclude  
 397 that a combination of  $K^*(800)$ ,  $K^*(892)$  and  $K^*(1410)$  provides a good description of the data. A  
 398 fit without the  $K^*(800)$ , i.e., using only a  $K^*(892) + K^*(1410)$  combination, can also provide a  
 399 good description of the data if the modelling of the  $\tau \rightarrow K_S^0 K_L^0 \pi^- \nu_\tau$  were to be incorrect by a  
 400 significant amount. The shape and rate distortions necessary for a two resonance model to fit the  
 401 data are however reasonably consistent with the large experimental uncertainties on the ALEPH  
 402 data that was used in the generation of this background mode. As such further study is necessary  
 403 on this mode and is ongoing.

404 Figure 10 shows the results of various measurements that went into calculating the 2007 PDG  
 405 average values for the mass and width of the  $K^*(892)$ . The Belle 2007 result and our result both  
 406 indicate a shift towards 895 MeV for the mass value.

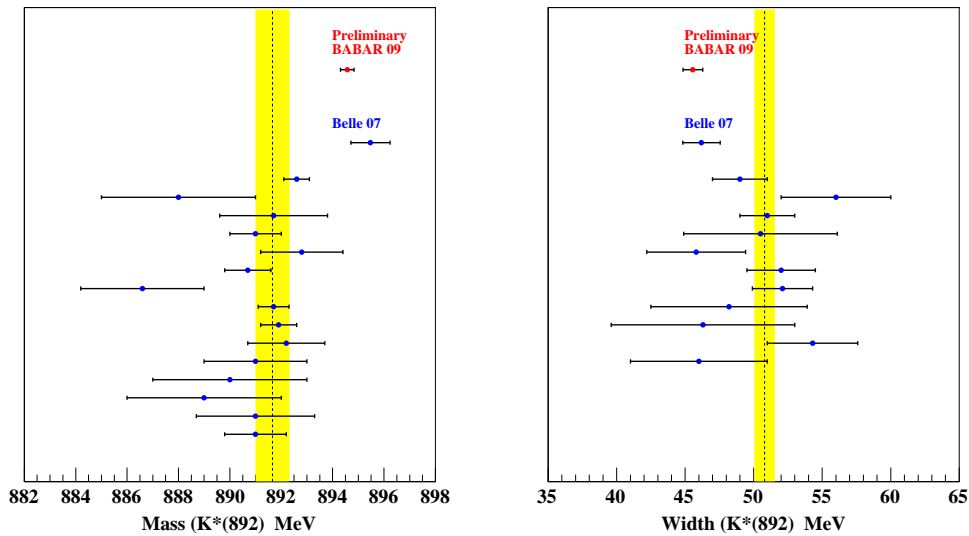


Figure 10: Comparison of the  $K^*(892)$  mass and width values which were included in the PDG07 calculated average value and the recent result from Belle, and our result. The majority of the PDG07 values are from hydrogen bubble chamber experiments.

## 7 ACKNOWLEDGEMENTS

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support *BABAR*. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the US Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), the Commissariat à l’Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung and Deutsche Forschungsgemeinschaft (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Foundation for Fundamental Research on Matter (The Netherlands), the Research Council of Norway, the Ministry of Education and Science of the Russian Federation, Ministerio de Educación y Ciencia (Spain), and the Science and Technology Facilities Council (United Kingdom). Individuals have received support from the Marie-Curie IEF program (European Union) and the A. P. Sloan Foundation.

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